The Effect of Non-Uniform Temperature Gradients on the Onset of Rayleigh-Bénard Convection in a Nanofluid

A Dissertation Submitted in Partial Fulfillment of the Requirements for the Award of the Degree of

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by

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Approval of Dissertation

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DECLARATION

I, Divya Zacharias, hereby declare that the dissertation, titled **The Effect of Non-Uniform Temperature Gradients on the Onset of Rayleigh-Bénard Convection in a Nanofluid** is a record of original research work undertaken by me for the award of the degree of Master of Philosophy in Mathematics. I have completed this study under the supervision of Dr S. Pranesh, Professor, Department of Mathematics.

I also declare that this dissertation has not been submitted for the award of any degree, diploma, associateship, fellowship or other title. It has not been sent for any publication or presentation purpose. I hereby confirm the originality of the work and that there is no plagiarism in any part of the dissertation.

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CERTIFICATE

This is to certify that the dissertation submitted by Divya Zacharias (Reg. No. 1435302) titled **'The Effect of Non-Uniform Temperature Gradients on the Onset of Rayleigh-Bénard Convection in a Nanofluid'** is a record of research work done by her during the academic year 2014 - 2016 under my supervision in partial fulfillment for the award of Master of Philosophy in Mathematics.

This dissertation has not been submitted for the award of any degree, diploma, associateship, fellowship or other title. It has not been sent for any publication or presentation purpose. I hereby confirm the originality of the work and that there is no plagiarism in any part of the dissertation.

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Abstract

Rayleigh-Bénard Convection in nanofluids is considered in this research work and the impact of non-uniform temperature gradients on the onset of convection is studied.The Galerkin technique is used to arrive at the Rayleigh number and wave number.The problem is analyzed under three different velocity boundary combinations namely free-free, rigid-free and rigid-rigid boundaries with isothermal, iso-nano concentration conditions and the eigenvalue is obtained for each of these cases.A linear stability analysis is performed using normal mode method. The influence of several parameters on the onset of convection has been investigated. One linear and five non-linear temperature profiles are employed and their comparative impact on onset of convection is summarized.

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Chapter 1

Introduction

1.1 Objective and Scope of the Dissertation

The science that deals with the behavior of fluids in motion or at rest and the correlation of fluids with solids or other fluids along the boundaries is characterized as Fluid Mechanics.This branch of science has varied applications and is widely used in everyday actions as well as in the designing of the advanced engineering systems.

An average house is an exposition hall filled with fluid mechanics applications.The plumbing methods for cold water, natural gas and sewage as well as refrigerators, heating and air conditioning systems are also modeled on the basis of fluid mechanics.Automobile industry also makes use of several efforts of fluid mechanics. All mechanisms connected with carriage of fuel from fuel tank to the cylinders are reviewed using concepts of fluid mechanics. In addition to these hydraulic brakes, power steering, lubrication systems, cooling systems and even the tires are customized on the basis of this branch of science. Various studies and observations have concluded that fluid mechanics methods play a major role in evaluation and invent of aircraft, rockets, submarines, biomedical devices, buildings, bridges etc.

The circulation system of the body is functioning as a fluid system. Artificial hearts, heart-lung machines, breathing aids and other such methods are dependent on the basic fluid mechanics standards. Various natural wonders such as rain cycle, weather patterns, winds, ocean waves, currents in large bodies and rise of ground water to the top of trees are also conducted by ideals of fluid mechanics.

Due to the relevance in many natural and industrial applications, much progress has been made in understanding the idea of convection in fluid mechanics over the last century. Convection is the joint, concerted movement of cluster of molecules within fluids through combination of advection and diffusion. In addition convection is an important means of transferring mass in fluids.The difference in temperature leads to density variation in fluids which give rise to natural convection. Bulk fluid movement occurs as cooler and denser particles fall down and sink while hotter and lighter components on the base layer rise up.

Rayleigh-Bénard convection is a kind of natural convection, in which a particular pattern of convection cells are developed by the fluid and it is named as Bénard cells. And this type of convection occurs due to the heating of a plane horizontal fluid layer from below. As a result of its analytical and the experimental relevance, the Rayleigh Bénard convection is one of the highly recommended convection phenomena.

In 1900, Henri Bénard [5], a French physicist, carried out an experiment for the first time through which the features of Rayleigh Bénard convection can be obtained.The investigational arrangement consists of a layer of liquid between two planes which are parallel to each other. On comparing with the dimension of horizontal layer, the height is small. In the beginning the temperatures of both the bottom plane and the top plane are the same. The temperature of the liquid will be same as that of its surrounding and hence the liquid will lead towards equilibrium. Gradually, thermal energy starts conducting through the fluid since there is a slight increase in temperature of the bottom plane. This results in density increase at the top layer than that of underneath layer. Finally, the system is heavy at the bottom but convective motion does not necessarily sets in because thermal diffusivity and viscosity of the fluid will oppose convective movement. If the fluid is adequately heated, then convective motion ensues steadily. The microscopic random movement of fluid particles will impulsively become well arranged on a macroscopic level, after the convection in established and as a result hexagonal Bénard convection cells will be formed.

Rayleigh [33] in 1916 investigated the theory for the condition for instability incorporating two free surfaces. Gravity acts on the fluid and tries to pull the cooler, denser liquid from the top to the bottom, since there is density gradient between both the plates. On the contrary the viscous damping force in the fluid opposes the gravitational force. Thus a non-dimensional parameter known as Rayleigh number is used to calculate the balance of these two forces and is given by,

$$R = \frac{\beta g \rho_0 d^3 \Delta T}{\mu \kappa}$$

where, β is the coefficient of thermal expansion, g is acceleration due to gravity, d is the distance between the plates, ΔT is the difference in temperature, μ is co-efficient of viscosity and κ is thermal conductivity.

It is summarized that the gravitational forces become more prevailing as the Rayleigh number increases. When the critical Rayleigh value is approached, the instability occurs and the convection cells emerge. As convection takes place for lower values of temperature gradients, studies reveal that it has got wide several applications in varied sectors such as bio-chemical reactions, environmental science, geophysics, manufacturing processes and so on.

Nanofluids, are the common heat conducting fluids containing nanometer sized(typically less than 100 nm) particles suspended in them. Materials, such as oxide ceramics (Al2O3, CuO), nitride ceramics(AlN, SiN), carbide ceramics (SiC, TiC), metals (Cu,Ag, Au), semiconductors (TiO2, SiC) and carbon nanotube are used as nanoparticles in carrier fluid. Liquids like water, ethylene glycol and oil have been used as base liquids in nanofluids. A new dimension is opened for an efficient way of advancing the heat transfer performance of common fluids using these nano particle suspensions.

The constantly increasing requirement of heat removaling techniques involving electronic chips, laser applications or other high energy devices has challenged the efficiency of heat transfer techniques. The conventional fluids put a fundamental limit on heat exchange processes due to their poor thermal conductivity. Several factors hinder the efficiency of usual techniques of heat transfer. Comparative studies on heat conducting competence show that the customary heat transferring liquids are less efficient than metals.

The idea of increasing transfer of thermal energy in fluids by dispersing millimeter or micrometer-sized particles in liquids was carried out by Maxwell [22] as well as various other scientists and engineers for more than a century. On the contrary, a number of drawbacks were exhibited by these slurries with micro to millimeter sized suspensions. Primarily, the tendency of these particles to rapidly settle in fluids under gravity brings fouling and several other issues. Another major problem in using such large particles is the increase in the pumping power. Furthermore, these suspensions can clog the tiny channels and small flow passages of the emerging miniaturized devices. In addition to these, erosion of components occurs due to the abrasive action of the particles and their momentum transfer requirement increases the pressure drop significantly. Hence it is observed that from the view of technological considerations these slurries cannot be used as heat transfer fluids even though they have higher conductivities.

Finally, it was observed that the disadvantages of slurries with micron or bigger size particles can be solved using nanometer dimensioned particles. Also the production of nano sized particles is made possible using recent progress in nanotechnology and associated manufacturing procedures. The Argonne National Laboratory conducted a series of research works and materialized this concept and Choi [12] was the first person to name the fluids with nanometer sized suspensions as nanofluids.

This discovery of nanotechnology-based fluids with improved heat transfer characteristics is really significant because these fluids display thermal properties finer to those of their base fluids or usual particle fluid suspensions. The advantage of using nano particle suspended fluids is that there are no issues of clogging and increase in pressure drop due to the miniature particle size and their small volume fraction. As a result of the large surface area of the nanoparticles the non equilibrium effect between fluid and solid as well as the sedimentation problem has reduced whereas the stability of the particles is increased. Therefore by reducing the particles to nanometer dimensions the troubles of customary slurries can be eradicated. These features have made nanofluids potential for cooling application such as energy intensive laser and X-ray applications, super conducting magnets, high speed computing systems, fiber manufacturing processes and high-speed lubrication applications.

The non uniform temperature profile finds its source in transient heating or cooling along the boundaries so that the basic temperature gradient is explicitly dependent on position and time. Few applications of nonuniform temperature gradient can be observed in geophysics due to the source of heat and due to the non uniform heating imparted by the sun to the several latitude zones around the world as well as oceanic and continental surfaces. Majority of the industrial problems consist sudden heating or cooling leading to non uniform temperature gradients.

The number of studies conducted in convective heat transfer area is restricted compared to that in the area of thermal conductivity of nanofluids. Hence,the main intent of the study is to examine the effect of non-uniform temperature gradients on the onset of Rayleigh-Bénard convection in nanofluids.

Chapter 2

Literature Review

The literature relevant to the theme of the dissertation is briefly reviewed below.The aim of this survey is to highlight the background literature pertaining to the topic under discussion. The literature related to nanofluids is discussed in following paragraphs.

The heat transfer characteristics of fluids containing nanosize particles were studied by Putra *et al.*[31] They placed a cylinder horizontally and experimented the onset of convection by heating and cooling the cylinder at opposite ends. Nanofluids exhibited apparently complex nature on examining the declination of conduct of heat in them. Several studies on finding the cause of deterioration led to the conclusion that various factors like the material of the nanoparticle, concentration of these particles , geometry of cavity etc play a vital role.

Choi *et al.*[13] studied the reformed Maxwell model and gave an explanation on the drawback of using spherical shaped particles. Consecutively to understand the effect of liquid/ solid interface, the Hamilton Crosser representation was broadened for non-spherical shaped particles. Along with this, a new model was suggested for three-phase suspension, which is formulated in terms of the thermal conductivity, concentration and experimental shape factor. Also the thermal conductivity limit on the basis of experimental shape factor is clarified by renovated Hamilton Crosser model.

Buongiornos [10] studies dealt with the seven slip mechanism in nanofluids and in turn the convection in them. Among the various slip mechanisms, Brownian diffusion and thermophoresis are of great relevance.For momentum,mass and heat transport,he formulated a four equation, double-component, non-homogeneous equilibrium model on the basis of his observations . A comparative study was also undertaken among nanoparticle and time and length of turbulent eddies ,which gave a clear view that convection of nanoparticles along with the fluid is homogeneous when turbulent eddies are present. As an impact of this,the turbulence intensity is vague. For the strange increment in values of heat transfer coefficient in nanofluid a detailed alternative clarification was given. He made a finding that nanofluid characteristics vary drastically between the layers of boundary as an impact of thermophoresis and temperature gradient. Because of these effects viscosity decreases between boundary layer for the heated fluid and thus the heat transfer will be enhanced. In order to give an explanation to these effects he also created a correlation structure.

Tzou [40] gave an overview on the nanofluids thermal instability and the nature of Brownian motion as well as thermophoresis. An observation was made that the nanofluids critical Rayleigh number value is less in magnitude than that of a regular fluid. He found that on adding highly promoted turbulence, nanofluids energy bearing capacity will raise, which in turn leads to higher heat transfer coefficient. Making use of non dimensional analysis the dominating groups are extracted. Additionally the methods of weighted residual and eigen function expansion were made use to obtain approximate solution of Rayleigh number.

Nield and Kuznetsov [23] examined nanofluid layer saturated with Porous medium and analytically observed the convection in them using a model developed on the effects of thermophoresis and Brownian diffusion. For studying the influence of Porous medium they employed Darcy model. To analyze the outcome of LTNE between the solid matrix, fluid and particle phases, a three- temperature model was taken into account. The conclusion obtained was that the local thermal non-equilibrium (LTNE) has minimal impact when they considered dilute nanofluids unlike certain other cases where this effect is important.

An analytical approach was used by Nield and Kuznetsov [24] in porous nanofluid layer to observe the convection onset. For this model of nanofluid, thermophoresis and brownian diffusion effects are used. In this work the Brinkman model for Porous medium is considered. Several observations were made by them as follows: for usual nanofluid (with high values of Lewis number), the buoyancy force along with nanoparticle conservation equations is the cause of main effect and also the impact of nanoparticles on the equation of thermal energy is because of the effect of second order. Another conclusion was made that ,an increase or decrease in critical Rayleigh number value can be achieved by a significant change in nanoparticles distribution i.e. the value will vary if the nanoparticle concentration is heavy at top or heavy at bottom . Another important finding was that if the nanopaticle distribution is bottom heavy then convection will set as oscillatory motion.

Wen *et al.*[42] produced an overview on the applications of nanofluids in the area of heat transfer based on analytical study. Studies were also done on the drawbacks of nanofluids, because eliminating these demerits can help in attaining progress and growth in this area.

Wong and Leon [43] have highlighted the remarkable range of applications of nanofluids which will make it highly demandable in current and future scenario. In their studies the main concern is on the controllable and extremely improved heat transfer characteristics of these fluids. Also the distinctive properties are highlighted such that they can be used for advanced applications.

Nield and Kuznetsov [25] considered how the theory of double diffusion has an impact on the convection onset in nanofluids with saturated porous media. For carrying out their studies they made use of binary fluid that is, salt water was chosen to be the base fluid. To study the effect of porous media Darcy model is applied. The equations on thermal energy constitute regular as well as cross diffusion terms. For understanding the oscillatory and non-oscillatory cases, the Galerkin technique was incorporated.

Yadav *et al.*[44] carried out an experiment on Rayleigh-Bnard convection in nanofluids and noticed the linear stability. On the contrary due to the effects of thermophoresis and Brownian motion, they had to bring in the nonlinear motion in the heat transfer in nanofluid.

Consecutively to obtain the exact solution of a layer of nanofluid confined between free-free boundaries the Galerkin technique was applied. Also the calculations using numerical methods were applied to figure out the value of critical Rayleigh number and plots of graphs were also obtained to give clarification to the impact of various parameters. They were also able to attain the situations that lead to over stability.

Nield and Kuznetsov [26] observed porous layer of nanofluid and made an analytical investigation on vertical flow consequence on the commencement of its convection. Their observations helped in comprehensing the dependency of critical Rayleigh number on factors such as Brownian motion and thermophoresis. This study was undertaken for both oscillatory and non-oscillatory cases,including and excluding through flow.

Bhadauria and Agarwal [6] considered a porous layer of nanofluid with coriolis force and observed the linear and nonlinear instability while conducting thermal energy. In the momentum equation,they regarded the Brinkman term as well as the term due to Coriolis force and also integrated the effects due to Brownian motion and thermophoresis. For carrying out linear and nonlinear studies normal mode method and Fourier series with truncated representation were incorporated respectively. Oscillatory as well as stationary modes of convection were experimented. Analyzing the weak nonlinear method the thermal as well as concentration Nusselt numbers were calculated. The nature of concentration and thermal Nusselt numbers are defined by finite amplitude equations and various numerical methods are implied to find the solutions for them. Lastly the outcomes are represented graphically.

The linear and nonlinear cases of onset of convection in porous nanofluids was examined by Bhadauria *et al.*[7].The equation of momentum used in this study constituted Brinkman model. Effects of Thermophoresis as well as Brownian motion were taken into consideration. Normal mode technique and truncated representation of Fourier series were performed for linear analysis and nonlinear study respectively. To visualize the impact of several parameters on convection onset graphs were depicted. The weak nonlinear analysis helped in formulating the concentration and the thermal Nusselt numbers. The equation of finite amplitude was evaluated using the numerical method and henceforth the thermal and the concentration Nusselt numbers' natures were summarized.

Bhadauria and Agarwal [8] observed the porous, rotating layer of nanofluid using thermal non-equilibrium model and studied the linear effect as well as nonlinear impact on the commencement of convection. The properties of porous medium was summarized with the help of Darcy model. Three types of temperature models were used for this study. Normal mode technique was employed for understanding linear stability and truncated representation of Fourier series substantiated the nonlinear theory.

Sheu [35] initialized the studies on viscoelastic nanofluid with porous medium and observed how the convection sets in. The constitutive equation of Oldroyd-B type is used in describing the flow pattern of nanofluids with viscoelastic effect. The effect of several parameters on the system stability was considered in depth. An observation was made by him that in both top heavy and bottom heavy distribution of nanoparticles the oscillatory mode can occur. An important conclusion was that unlike stationary mode several processes like Brownian diffusion, viscoelasticity and so on triggers the convection to commence in oscillatory mode.

Timofeeva *et al.*[39] examined a complicated nanofluid system and presented the systematic theory associated with it. A system of three phases such as solid phase, interfacial phase and liquid phase constituted,the suspension of nanoparticle .These phases gave considerable contributions to the properties of system since it has enormously great ratio of surface-to-volume. Additionally the impact of several nanofluid parameters in heat transfer was also estimated in this article.

Yu and Xie [47] recapitulated the advance in the study of nanofluids till then as follows; preparation method, evaluation methods for stabilizing nanofluid system and mechanisms to improve the nanofluids stability. The immense range of nanofluids applications in diverse fields are also mentioned in the article. Eventually various prospects for research in nanofluids are also suggested.

Saidur *et al.*[34] has summarised the remarkable range of applications of nanofluids as well as has posed the various challenges in this field of research. On the basis of past literature they reviewed the nanofluids thermal conductivity and observed that it is very high. Also they found that on comparing with regular fluid nanofluids exhibit strong dependence on temperature when particle concentration is minimal.This property is considered as the prime factor for the immense applications of nanofluids in current scenario. Meanwhile this paper also mentions few industrial challenges that are hindering the usage of nanofluids.

Nield and Kuznetsov [27] performed the linear stability theory in a porous nanofluid layer and extended the same for the Horton-Rogers-Lapwood problem. They observed the dependency of viscosity and thermal conductivity of the nanofluid on the nanoparticle concentration. The impact of thermophoresis and Brownian motion was also employed. An observation was made that the nanoparticle concentration is stratified when Brownian motion is associated with it and as a result thermal conductivity and viscosity also gets stratified. They took a sample of dilute nanoparticles and hence they treated the porous layer of fluid as weak heterogeneous layer that will vibrate perpendicularly to viscosity and conductivity. This technique helped in obtaining the analytical solution approximately.

The features of double- diffusive convection in a porous layer of nanofluid was summarized by Agarwal *et al.*[1]. Nanofluid was prepared making use of binary fluid. The proposed model for studying nanofluids included Brownian diffusion and thermophoresis effects. Thermal energy equation was composed of cross diffusion and diffusion terms. The technique of normal mode was performed to understand linear stability and the nonlinear theory was comprehensed using truncated representation of Fourier series. Analysis of linear theory was done by calculating critical Rayleigh number and investigation of non linearity was based on Nusselt number.

The internal heating impact on the onset of the Darcy Brinkman convection in a layer of porous nanofluid was carried out by Yadav *et al.*[45]. The BrinkmanDarcy model based equation characterized the nanofluid motion, under the condition that viscosity of fluid varies from effective viscosity. To obtain the solution of eigenvalue problem linear theory was considered and Galerkin method was employed. Graphs were plotted to show how different parameters affect the stability of the system.

Mahian *et al.*[21] examined how nanofluids can be applied in solar thermal systems. For the conservation of environment and since fossil fuels are non-renewable, these days researchers are motivated to utilize the solar energy as an alternate energy source. Through their study these researchers have mentioned advantages like the effect of nanofluid on various solar devices. They also investigated the application of nanofluids in solar cells, thermal storage system, photovoltaic system and so on.Meanwhile the challenges faced by nanofluids in this area were also taken into consideration.

Yadav *et al.*[46] studied various properties of the convection in a layer of electrically conducting nanofluid under the application magnetic field and considered linear theory for the study. The model was inclusive of the effects of Brownian diffusion as well as thermophoresis.For free-free, rigid-rigid and rigid- free boundaries the analytical solution of eigenvalue problem is formulated. Galerkin technique was used for obtaining numerical solution. Their experiments were conducted on aluminium water nanofluid and they presented the numerical solution .

Gupta *et al.*[14] investigated convection in a layer of nanofluid with vertical magnetic field and expressed the solution using linear stability theory. An assumption was made that the nanoparticles are distributed as bottom heavy. An explaination was given on system stability and was proposed that the system is more stable when the mode is oscillatory rather than stationary, The density gradient fluctuates with the variation of density,this is due to the bottom heavy distribution of nanoparticles and also it occurs due to bottom heating of nanofluid. They summarized that the buoyancy effect coupled with nanoparticles conservation lead to phenomenon of instability and thermophoresis and Brownian motion to the thermal energy equation have no contribution at all. Both the oscillatory and stationary convections were analysed using normal mode technique. And the stabilizing effect of magnetic field on both convective modes was noticed.

Mahajan and Arora [19] employed linear theory on a layer of magnetic nanofluid with coriolis force for understanding the convection in it. The model developed by them to study the features of nanofluid included the influence of magnetophoresis, Brownian diffusion and thermophoresis. The eigen value problem is solved using Chebyshev Pseudospectral method. The result is obtained for the following boundary conditions: rigid-rigid, free-free, and rigid-free. They treated ester and water based magnetic nanofluids. In this paper they have investigated how the onset of convection is affected by rotation, magnetic field, and modified particle density increment . An interesting observation was made that the magnet mechanism will overcome the buoyancy mechanism if the thickness of fluid layer is 1mm. It was also noticed that in an environment of microgravity the magnetic nanofluids are more flexible on the onset of convection. They summarized for all the three boundary conditions, the temperature gradients should be high so that the convection will start.

Agarwal and Bhadauria [2] analysed characteristics of Newtonian nanofluids. Linear as well as non linear stability was examined. The significant growth in the value of critical Rayleigh number for linear theory was observed if the nanoparticle distribution is bottom heavy. In addition to this weak nonlinear theory provided the thermal and concentration Nusselt numbers. A comparative study was undertaken between nanofluid and binary fluid convections by introducing Soret effect for both cases.

Bhadauria and Kiran [9] observed heat transfer in nanofluids and formulated a mathematical model to manage the heat transfer in them. The five mode Lorenz model was used in evaluating weak nonlinear theory. They obtained the expression for Nusselt number. They noticed the dependency of transport of the heat and the convection modulation. On observing the effect of internal Rayleigh number the finding was that there should be increase in nanoparticle and heat transfer.

Rana *et al.*[32] investigated double-diffusive convection in porous layer of nanofluid under coriolis force. Studies on porous medium was carried out using Darcy model. Binary fluid such as salty water was treated as base fluid for the experiments on nanofluids. A finding was made that the solute gradient and rotation has stabilizing effect on stationary convection. When thermal stability of nanofluid layer was reviewed, it was analysed that rotation has a major role. This is made use in machineries with rotating force.

The influence of rotation on the onset of thermal instability in a layer of Newtonian nanofluid was examined by Agarwal and Bhadauria [3].For understanding the unsteady state ,non-linear analysis was employed and thus evaluated the thermal and concentration Nusselt number values .They noticed that the rate of heat and mass transfer is stabilized by rotation and further the onset of convection is also stabilized. The findings were summarized as follows that the onset of convection is delayed for rotating system on comparing with nonrotating system.

Mahajan and Sharma [20] investigated magnetic nanofluid with less permeability and noticed the onset of convection in it. The consequences of magnetophoresis, Brownian diffusion, and the thermophoresis on nanofluids were evaluated using this model.The Chebyshev pseudospectral method provided the solution for eigenvalue problem. The results were formulated for various boundary surface conditions. In addition to this, experiments were conducted to summarize how various parameters like permeability, modified particle density increment and magnetic field affect the stability of system under consideration. On examinining high flexibility was displayed by magnetic nanofluid at the onset of convection. Finally, it was presented that all types of boundary conditions require high value of temperature gradient is required such that the convection starts.

Umavathi [41] conducted a study on porous layer of nanofluid under the effect of modulating temperature. Three types of temperature modulation were set at the boundaries.The perturbation method with small amplitude was employed for evaluating the wave numbers as well as the critical Rayleigh number.Also on investigation it was noticed that if the wall temperature is modulated periodically the system can be stabilized or destabilized . The analysis led to the fact that the stabilizing effect is comparatively more for nanofluids than regular heat transfer fluid.

Kiran [18] summarized the properties of the thermal convection in the porous, viscoelastic nanofluid under vibration effect. The constitutive equation of the Oldroyd-B type were proposed for comprehensing the rheological natureof viscoelastic nanofluid. The non-uniform vertical vibrations of the system was varying periodically with time,when the system was oscillating vertically. Inorder to investigate the heat and mass transfer truncated representation of Fourier series was incorporated to study nonlinear theory. The summary of the study was that inorder to organize heat and mass transport in the system the gravity modulation should be employed effectively .

Siddheshwar and Meenakshi [38] treated nanofluids as a single phase system and proposed a model for them after investigating the onset of RayleighBnard convection. Various properties were reviewed based on their studies. They formulated a tri-modal Lorenz model on presuming small scale convections and Boussinesq approximation . In addition to this the GinzburgLandau equation was also derived from the generalized Lorenz model. Using the convective modes amplitudes the heat transport was estimated and was presented analytically. An observation was made that the presence of nano size particles has enhanced the heat transport enhancement. Comparitive study was made to see the increment in heat transport in Newtonian nanofluids as that of regular Newtonian liquids.

Kakac and Pramuanjaroenkij [17] presented various benefits of producing thermal systems with nanofluids and highlighted the probablity of using it in heat transfer enhancement. Two categories are given emphasis in this paper. The first category is the one in which base fluid and nanoparticle is assumed to be single mixture and is termed as single-phase modeling. The other one is called as two-phase modelling in which nanoparticles characteristics and base fluids are treated separately.

Shivkumara and Dhananjaya [36] considered the porous, rotating nanofluid layer and observed the thermal convective instability. Along the concentration of nanoparticles they employed physically realistic boundary combinations. For numerical solution of generalized eigenvalue problem Galerkin technique was used. The values of Critical Rayleigh numbers, frequencies and wave numbers were estimated for checking over stability.

The literature pertaining to non-uniform temperature gradient is reviewed as follows.
Siddeshwar and Pranesh [37] employed one linear and five non-linear temperature gradients on a micropolar fluid system and investigated the convection onset. On applying Galerkin method they calculated the eigen value of the problem for various boundary surface combinations. In addition to this, the comparative study led to the observation that micropolar fluid Rayleigh number is more when compared to that of newtonian fluid.

In a micropolar fluid the impact of the six non-uniform temperature profiles on B$\acute{e}$nardMarangoni convection was summarized by Idrisa *et al.*[15]. Linear theory was considered by them to obtain the solution and hence they examined the functionality of several parameters on the fluid system. The highlight of the study was that by opting the appropriate temperature profile the convection can be managed.

Pranesh and Riya Baby [29] considered micropolar fluids subjected to electric field and reference state steady temperature profiles and highlighted their impact on Rayleigh-Bnard convection. They assumed that microrotation vanishes at the boundaries. Authors have substantiated how several fluid parameters and electric Rayleigh number affect the micropolar fluid system.

Pranesh and Arun Kumar [28] examined a layer of micropolar fluid constrained to double diffusive convection and they highlighted the effect of non-uniform basic temperature gradient on convective motion of this fluid layer. Using linear stability theory they presented in detail how various micropolar parameters have an impact on the onset. For carrying out the comparative study one linear and five non linear temperature profiles were employed and finally the results were graphically depicted. The study was summarized that the fluid containing suspensions ,heated and soluted from below exhibited high stability in comparison with the classical fluid without suspended particles.

Joseph *et al.*[16] performed Galerkin technique to obtain the solution for the problem on the onset of Rayleigh-Bnard-Marangoni convection in micropolar fluid subjected to electric field and non-uniform basic temperature gradient. They proposed linear stability analysis for carrying out this study.Micropolar fluid parameters effects and electric Rayleigh number impacts were observed . Varied non-uniform temperature profiles are reviewed and their comparative influence on onset is evaluated.

Chapter 3

Basic Equations, Boundary Conditions, Approximations and Dimensionless Parameters

The governing equations, assumptions, boundary conditions and the dimensionless parameters considered in this problem are discussed in this chapter.(See Ritu [4])

3.1 Basic Equations

The governing equations associated with the problem are as follows:

Conservation of mass:

$$\frac{\partial \rho}{\partial t} + \bigtriangledown \cdot (\rho \vec{q}) = 0 \tag{3.1}$$

For an incompressible fluid, ρ is constant,from equation (3.1),we get

$$\bigtriangledown \cdot \vec{q} = 0 \tag{3.2}$$

Conservation of Linear Momentum:

The Navier-Stoke's equation under the Boussinesq approximation is,

$$\rho_f[\frac{\partial \vec{q}}{\partial t} + (\vec{q}\cdot\bigtriangledown)\vec{q}] = -\bigtriangledown p + \mu \bigtriangledown^2 \vec{q} - \rho \vec{g} k$$

For a small volumetric fraction Φ,the nanofluid density is given as,

$$\rho = \Phi\rho_p + (1-\Phi)\rho_f$$

On using the equation of state, $\rho_f = \rho_{0f}\,[1-\beta(T-T_1)]$,we get,

$$\rho_f[\frac{\partial\vec{q}}{\partial t} + (\vec{q}.\bigtriangledown)\vec{q}] = -\bigtriangledown p + \mu \bigtriangledown^2 \vec{q} - [\Phi\rho_p + (1-\Phi)\rho_f(1-\beta(T-T_1))]\vec{g} \tag{3.3}$$

Conservation of Energy:

The heat transport equation for the nanofluid is,

$$\rho_f c_f[\frac{\partial T}{\partial t} + (\vec{q}.\bigtriangledown)T] = \kappa \bigtriangledown^2 T + \rho_p c_p[D_B(\bigtriangledown\Phi)(\bigtriangledown T) + \frac{D_T}{T_1}(\bigtriangledown T)^2] \tag{3.4}$$

Conservation of Nano Particle:

In the absence of chemical reaction,conservation of nanopaticle can be written as,

$$\frac{\partial\vec{\Phi}}{\partial t} + (\vec{q}.\bigtriangledown)\vec{\Phi} = D_B \bigtriangledown^2 \vec{\Phi} + \frac{D_T}{T_1} \bigtriangledown^2 T \tag{3.5}$$

3.2 Approximations

The following assumptions are made in solving this problem.

1. The fluid is incompressible and homogeneous.

2. The Boussinesq approximation.

3. The gravity acts vertically downwards.

4. Thermal diffusivity and viscosity are assumed to be constants.

5. There are no chemical reactions and negligible external forces in the nanofluid.

6. The nanofluid under consideration is a dilute mixture ($\phi << 1$).

7. Local thermal equilibrium are assumed between base fluid and nanoparticles.

3.3 Boundary Condition

3.3.1 Boundary Conditions on Velocity

To obtain the boundary conditions on velocity, Cauchy's Stress Principle,the no slip conditions and mass balance are considered and they are also dependent on the nature of the surfaces at boundary of the fluid,whether they are free or rigid.In this problem the following boundary surfaces are employed.

1. Both the boundaries of the fluid are free.

2. Lower boundary is rigid and upper boundary is free.

3. Both the boundaries of the fluid are rigid.

The velocity boundary conditions of free surfaces at the boundaries are

$$w = \frac{\partial^2 w}{\partial z^2} = 0. \tag{3.6}$$

The velocity boundary conditions of rigid surfaces at the boundaries are

$$w = \frac{\partial w}{\partial z} = 0. \tag{3.7}$$

3.3.2 Boundary Conditions on Temperature

The temperature boundary conditions are formulated on the basis of the heat conducting property of the boundaries.

Uniform and time independent temperature is observed if thermal capacity is large and thermal conductivity is high.Hence along the boundary surfaces the condition on temperature is given by

$$T = 0 \tag{3.8}$$

This condition is known as isothermal boundary condition or fixed surface temperature condition.

3.3.3 Boundary Conditions on Volumetric Fraction of Nanoparticles

The boundary conditions on volumetric fraction of nanoparticles are defined as follows

$$\left.\begin{aligned} &\phi = \phi_0, at z = 0 \\ &\phi = \phi_1, at z = d \\ &where, \phi_0 > \phi_1 \end{aligned}\right\} \tag{3.9}$$

$\phi_0 > \phi_1$ represents the bottom heavy condition.

3.4 Dimensionless Parameters:

The following are the dimensionless parameters that occur in this problem:

1. **Rayleigh number:**

$$R_a = \frac{\rho_0 \beta (T_0 - T_1) g d^3}{\mu \kappa}$$

Rayleigh number gives the relation between the buoyancy and dissipative force of viscosity and thermal conductivity. The convection sets in when Rayleigh number is greater than certain critical value.

2. **Prandtl Number:**

$$Pr = \frac{\mu}{\rho_0 \kappa}$$

Prandtl number is the ratio of kinematic viscosity to thermal diffusivity.

3. **Basic Density Rayleigh Number:**

$$R_m = \frac{[\rho_p \phi_1 + \rho_c (1 - \phi_1)] g d^3}{\mu \kappa}$$

Basic density Rayleigh number is the ratio of density of the nanofluid to the dissipation force of viscous and thermal.

4. **Concentration Rayleigh Number:**

$$R_n = \frac{(\rho_p - \rho_c)(\phi_0 - \phi_1)gd^3}{\mu\kappa}$$

Concentration Rayleigh number is the ratio of the volumetric fraction of nanoparticle to the dissipation force of viscous and thermal.

5. **Modified Diffusivity Ratio:**

$$N_A = \frac{D_T(T_0 - T_1)}{D_B T_1(\phi_0 - \phi_1)}$$

It is the ratio of thermophoretic diffusion coefficient to Brownian diffusion coefficient.

6. **Modified Particle Density Increment:**

$$N_B = \frac{\rho_p C_p(\phi_0 - \phi_1)}{\rho_f C_f}$$

It is the ratio of specific heat of particle to the specific heat of fluid.

7. **Lewis number:**

$$L_e = \frac{\kappa}{D_B}$$

The Lewis Number (Le) is defined as the ratio of the Schmidt Number (Sc) and the Prandtl Number (Pr).It is also the ratio of thermal diffusivity and molecular diffusivity.

Table 3.1: Nomenclature

Symbol	**Meaning**
d	Distance between the plates
T	Temperature
$\vec{q}$	Velocity
p	Pressure
μ	Co-efficient of viscosity
$\vec{g}$	Acceleration due to gravity
ρ_f	Fluid density
ρ_p	Nanoparticle mass density
ρ_0	Density at reference temperature
D_B	Brownian diffusion coefficient
D_T	Thermophoretic diffusion coefficient
C_f	Specific heat of nanofluid
C_p	Specific heat of nanoparticle
κ	Thermal conductivity
β	Co-efficient of thermal expansion
ϕ	Nanoparticle volume fraction
ϕ_0	Nanoparticle volume fraction at lower plate
ϕ_1	Nanoparticle volume fraction at upper plate
t	Time
T_0	Temperature at the lower plate
T_1	Temperature at the upper plate
ΔT	Temperature difference between both the plates
l, m	wave numbers in horizontal directions where $a^2 = l^2 + m^2$
∇	Vector Differential Operator
∇^2	Three dimensional Laplacian Operator

Chapter 4

The Effect of Non-Uniform Temperature Gradients on the Onset of Rayleigh-Bénard Convection in a Nanofluid

In this chapter,the effect of the non-Uniform temperature gradients on the onset of Rayleigh-B$\acute{e}$nard convection in nanofluids is studied.The linear analysis is done using normal mode analysis and the eigen values are obtained by Galerkin procedure.

4.1 Mathematical Formulation

A nanofluid layer is considered between two parallel surfaces of infinite length, separated by a distance d,as shown in figure 5.1.In the analytical formulation all thermophysical nanofluid properties are treated as constants.Let the temperature at the lower and upper boundaries be denoted by T_0 and T_1 respectively where $T_0 > T_1$.Also the nanofluid under consideration is assumed to be incompressible.

The governing equations of the problem are: (See Ritu [4])

Continuity Equation:

$$\bigtriangledown.\vec{q} = 0 \tag{4.1}$$

Conservation of Linear Momentum:

$$\rho_f[\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\bigtriangledown)\vec{q}] = -\bigtriangledown p + \mu \bigtriangledown^2 \vec{q} - [\Phi\rho_p + (1-\Phi)\rho_f(1-\beta(T-T_1))]\vec{g} \tag{4.2}$$

Conservation of Energy:

$$\rho_f c_f[\frac{\partial T}{\partial t} + (\vec{q}.\bigtriangledown)T] = \kappa \bigtriangledown^2 T + \rho_p c_p[D_B(\bigtriangledown\Phi)(\bigtriangledown T) + \frac{D_T}{T_1}(\bigtriangledown T)^2] \tag{4.3}$$

Conservation of Nanofluid particle:

$$\frac{\partial \vec{\Phi}}{\partial t} + (\vec{q}.\bigtriangledown)\vec{\Phi} = D_B \bigtriangledown^2 \vec{\Phi} + \frac{D_T}{T_1} \bigtriangledown^2 T \tag{4.4}$$

The quantities in (4.1)-(4.4) are as defined in chapter 3.The basic equations are solved subject to the boundary conditions given below.At the boundaries the temperature as well as the volumetric fraction of the nano particles are supposed to be constants.

1. **Free-Free Isothermal Iso-nano concentration:**

$$\left.\begin{aligned} w = \frac{\partial^2 w}{\partial z^2} = 0, T = T_0, \phi = \phi_0, at z = 0 \\ w = \frac{\partial^2 w}{\partial z^2} = 0, T = T_1, \phi = \phi_1 . at z = d \end{aligned}\right\} \tag{4.5}$$

2. **Rigid-Rigid Isothermal Iso-nano concentration:**

$$\left.\begin{aligned} w = \frac{\partial w}{\partial z} = 0, T = T_0, \phi = \phi_0, at z = 0 \\ w = \frac{\partial w}{\partial z} = 0, T = T_1, \phi = \phi_1, at z = d \end{aligned}\right\} \tag{4.6}$$

3. **Rigid-Free Isothermal Iso-nano concentration:**

$$\left.\begin{aligned} w = \frac{\partial w}{\partial z} = 0, T = T_0, \phi = \phi_0, at z = 0 \\ w = \frac{\partial^2 w}{\partial z^2} = 0, T = T_1, \phi = \phi_1, at z = d \end{aligned}\right\} \tag{4.7}$$

4.2 Non-Dimensionalisation

The equations (4.1)-(4.4) are non-dimensionalised making use of following definitions.

$$\left.\begin{aligned} (x^*, y^*, z^*) = (\frac{x}{d}, \frac{y}{d}, \frac{z}{d}), t^* = \frac{\kappa t}{d^2}, p^* = \frac{d^2 p}{\rho_f \kappa^2} \\ \vec{q}^* = \frac{d\vec{q}}{\kappa}, \phi^* = \frac{\phi - \phi_1}{\phi_0 - \phi_1}, T^* = \frac{T - T_1}{T_0 - T_1} \end{aligned}\right\} \tag{4.8}$$

Substituting equation (4.8)into the governing equations and on neglecting asterisks the following non-dimensionalized equations are obtained.

$$\nabla.\vec{q} = 0 \tag{4.9}$$

$$\frac{1}{P_r}\left[\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\,\vec{q}\right] = -\nabla p + \nabla^2 \vec{q} - R_n \phi + R_a T - R_m \tag{4.10}$$

$$\frac{\partial T}{\partial t} + (\vec{q}.\nabla)\,T = \nabla^2 T + \frac{N_B}{Le}(\nabla\phi.\nabla T) + \frac{N_A N_B}{Le}(\nabla T.\nabla T) \tag{4.11}$$

$$\frac{\partial \phi}{\partial t} + (\vec{q}.\nabla)\,\phi = \frac{1}{Le}\nabla^2\phi + \frac{N_A}{Le}\nabla^2 T \tag{4.12}$$

The non-dimensional parameters $P_r, R_m, R_n, R_a, N_A, N_B, L_e$ in equations (4.9)-(4.12) are as defined in chapter 3.

The boundary conditions given by equations (4.5)-(4.7) in dimensionless form can be written as

1. **Free-Free Isothermal Iso-nano concentration:**

$$\left.\begin{aligned} w = D^2 w = 0, T = 0, \phi = 1, at z = 0 \\ w = D^2 w = 0, T = 0, \phi = 0, at z = 1 \end{aligned}\right\} \tag{4.13}$$

2. **Rigid-Rigid Isothermal Iso-nano concentration:**

$$\left.\begin{aligned} w = Dw = 0, T = 0, \phi = 1, at z = 0 \\ w = Dw = 0, T = 0, \phi = 0, at z = 1 \end{aligned}\right\} \tag{4.14}$$

3. **Rigid-Free Isothermal Iso-nano concentration:**

$$\left.\begin{aligned} w = Dw = 0, T = 0, \phi = 1, at z = 0 \\ w = D^2 w = 0, T = 0, \phi = 0, at z = 1 \end{aligned}\right\} \tag{4.15}$$

4.3 Basic State

The basic state of the fluid is assumed to be at rest and is given by

$$\vec{q} = 0, T = T_b(z), p = p_b(z), \phi = \phi_b(z), \frac{-d}{\Delta T}\frac{dT_b}{dz} = f\,(z) \tag{4.16}$$

On substituting equation (4.16) in the equations (4.9)-(4.12) we obtain the following equations,

$$-\frac{dP_b(z)}{dz} - R_n\phi_b(z) + R_a T_b(z) - R_m = 0 \tag{4.17}$$

$$\frac{d^2T_b}{dz^2} + \frac{N_B}{Le}\left(\frac{d\phi_b}{dz}.\frac{dT_b}{dz}\right) + \frac{N_A N_B}{Le}\left(\frac{dT_b}{dz}.\frac{dT_b}{dz}\right) = 0 \tag{4.18}$$

$$\frac{1}{Le}\frac{d^2\phi_b}{dz^2} + \frac{N_A}{Le}\frac{d^2T_b}{dz^2} = 0 \tag{4.19}$$

On using the order of magnitude analysis,the second and third terms in (4.18) are small and this can be discarded.Therefore,we have

$$\frac{d^2T_b}{dz^2} = 0 \tag{4.20}$$

And also on substituting (4.20) in (4.19),we obtain

$$\frac{d^2\phi_b}{dz^2} = 0 \tag{4.21}$$

On integrating equations (4.20) and (4.21) and on using boundary condition the following equations are obtained.

$$T_b(z) = 1 - z \tag{4.22}$$

$$\phi_b(z) = 1 - z \tag{4.23}$$

4.4 Linear Stability Analysis

On imposing infinitesimal perturbations on the basic state,we obtain

$$\vec{q} = \vec{q_b} + \vec{q}', p = p_b + p', T = T_b + T', \phi = \phi_b + \phi', \tag{4.24}$$

here,the quantities with prime are the perturbations and quantities with suffix b denote the basic state values.

Using equation (4.24) in the equations (4.9)-(4.12) and also using the solutions of basic state,the following linearised equations governing perturbed state are obtained.

$$\bigtriangledown.\vec{q}' = 0 \tag{4.25}$$

$$\frac{1}{P_r}\frac{\partial \vec{q}'}{\partial t} = -\nabla p' + \nabla^2\vec{q}' - R_n\phi' + R_aT' \tag{4.26}$$

$$\frac{\partial T'}{\partial t} - w'f(z) = \nabla^2 T' \tag{4.27}$$

$$\frac{\partial \phi'}{\partial t} - w' = \frac{1}{Le}\nabla^2\phi' + \frac{N_A}{Le}\nabla^2 T' \tag{4.28}$$

where $f(z)$ is the non-dimensional basic temperature gradient,which is non-negative and find its source in transient heating or cooling along the boundaries.The various non-uniform temperature gradients considered in this dissertation are defined below.

Table 4.1: Non-Uniform Temperature Gradients

Basic temperature Profiles	Model	$f(z)$
Linear	R_{c1}	1
Heating from below	R_{c2}	$\begin{cases} \frac{1}{\epsilon}, 0 \leq Z < \epsilon \\ 0, \epsilon < Z \leq 1 \end{cases}$
Cooling from above	R_{c3}	$\begin{cases} 0, 0 \leq Z < 1-\epsilon \\ \frac{1}{\epsilon}, 1-\epsilon < Z \leq 1 \end{cases}$
Step function	R_{c4}	$\delta(z-\epsilon)$
Inverted parabolic	R_{c5}	$2(1-z)$
Parabolic	R_{c6}	$2z$

Operating curl twice on (4.26) to eliminate pressure and considering only the z component,the following equation is obtained.

$$\left(\frac{1}{P_r}\frac{\partial}{\partial t} - \nabla^2\right)\nabla^2 w' = R_a\nabla_1^2 T' - R_n\nabla_1^2\phi' \tag{4.29}$$

where ∇_1^2 is the two dimensional laplacian operator.

4.5 Method of Solution

4.5.1 Normal Mode Analysis

To obtain solution of stationary convection and the unknown fields w',T',ϕ',we use normal mode analysis and obtain the following.(See Chandrasekhar [11])

$$\begin{bmatrix} w' \\ T' \\ \phi' \end{bmatrix} = \begin{bmatrix} W(z) \\ T(z) \\ \phi(z) \end{bmatrix} e^{i(lx+my)} \tag{4.30}$$

where l and m are horizontal wave number in x and y direction.

Using the equation (4.30) in equations (4.27),(4.28) and (4.29) we obtain,

$$(D^2 - a^2)^2 W = a^2 R_a T - a^2 R_n \phi \tag{4.31}$$

$$-Wf(z) = (D^2 - a^2)T \tag{4.32}$$

$$-W = \frac{1}{L_e}(D^2 - a^2)\phi + \frac{N_A}{L_e}(D^2 - a^2)T \tag{4.33}$$

where,$D = \frac{d}{dz}$ and $a^2 = l^2 + m^2$ is the dimensionless wave number.

4.5.2 Galerkin Technique

For solving the set of differential equations (4.31)-(4.33), Galerkin technique is applied.To obtain the solution,multiply equation (4.31) by W,equation (4.32) by T and equation (4.33) by ϕ and integrate the resultant equations with respect to z between the limits 0 and 1.

Also let $W = AW_1, T = BT_1, \phi = C\phi_1$ where W_1, T_1, ϕ_1 are trial functions satisfying the boundary conditions and A,B,C are constants.

On substituting and simplyfying we obtain the expression for Rayleigh number R_a.

$$R_a = \frac{1}{a^2 X_7}\left[\frac{-X_2 * X_1}{X_6} + \left(\frac{X_4 * X_2}{\frac{1}{Le} * X_5 * X_6} - \frac{N_A * X_3}{X_5}\right) * a^2 R_n X_4\right], \qquad (4.34)$$

Where,

$X_1 =< W_1(D^2 - a^2)^2 W_1 >$,

$X_2 =< T_1(D^2 - a^2)T_1 >$,

$X_3 =< \phi_1(D^2 - a^2)T_1 >$,

$X_4 =< \phi_1 W_1 >$,

$X_5 =< \phi_1(D^2 - a^2)\phi_1 >$,

$X_6 =< f(z)T_1 W_1 >$,

and $X_7 =< W_1 T1 >$.

The following boundary conditions are considered,to find the critical Rayleigh number .

1. **When both boundaries are free,isothermal and isonano concentration:**

   The boundary conditions are:

$$w = D^2 w = 0, T = 0, \phi = 0, at z = 0, z = 1 \qquad (4.35)$$

The trial functions satisfying the boundary conditions (4.35) are:

$$\left.\begin{aligned} W_1 &= sin\pi z, \\ T_1 &= z - z^2, \\ \phi_1 &= z - z^2 \end{aligned}\right\} \tag{4.36}$$

2. **When both boundaries are rigid,isothermal and isonano concentration:**

   The boundary conditions are:

$$w = Dw = 0, T = 0, \phi = 0, at z = 0, z = 1 \tag{4.37}$$

The trial functions satisfying the boundary conditions (4.37) are

$$\left.\begin{aligned} W_1 &= z^2(1-z)^2, \\ T_1 &= z - z^2, \\ \phi_1 &= z - z^2 \end{aligned}\right\} \tag{4.38}$$

3. **When upper boundary is free,isothermal and isonano concentration and lower boundary is rigid,isothermal and isonano concentration:**

   The boundary conditions are:

$$\left.\begin{aligned} w = Dw = 0, T = 0, \phi = 0, at z = 0 \\ w = D^2w = 0, T = 0, \phi = 0, at z = 1 \end{aligned}\right\} \tag{4.39}$$

The trial functions satisfying the boundary conditions (4.39) are

$$\left.\begin{aligned} W_1 &= z^2(1-z)(3-2z), \\ T_1 &= z - z^2, \\ \phi_1 &= z - z^2 \end{aligned}\right\} \tag{4.40}$$

On substituting the trial functions (4.36),(4.38) and (4.40) in equation (4.34) and integrating we obtain the required expression for critical Rayleigh number,which attains its minimum at a_c^2 for given $f(z)$.

Chapter 5

Results and Discussions

The impact of non-uniform temperature gradients on the onset of Rayleigh-B$\acute{e}$nard convection in nanofluids is investigated using linear stability analysis.Galerkin technique is used to obtain the eigen value of the problem and is a function of concentration Rayleigh number R_n,Lewis number L_e,modified diffusivity ratio N_A.The range of values of R_n,L_e and N_A used to calculate the critical Rayleigh number are taken according to Pranesh and Ritu Bawa [30].The following boundary combinations are considered in the present study.

1. Free-free,isothermal and iso-nano concentration.
2. Rigid-rigid,isothermal and iso-nano concentration.
3. Rigid-free,isothermal and iso-nano concentration

The criteria of stability is evaluated in terms of critical rayleigh number using linear stability theory.Below the critical Rayleigh number,the system is stable,wheras above this value the system is unstable and convection sets in.The results summarized in this problem are depicted in the figures (5.2)-(5.10)

One linear and five non-linear basic temperature profiles are used in this study.We find that $R_{c4} < R_{c2} < R_{c3} < (R_{c1} = R_{c5} = R_{c6})$ for symmetric boundaries and $R_{c4} < R_{c3} < R_{c2} < R_{c6} < R_{c1} < R_{c5}$ for non-symmetric boundaries. (Refer Table 4.1 for notations). It is observed that step function and linear are the most destabilizing and stabilizing profiles in the case of symmetric boundaries.Step function and inverted parabolic are the most destabilizing and stabilizing profiles in non-symmetric boundaries.

Figures (5.2)-(5.4)are the plots of Rayleigh number Ra versus concentration Rayleigh number Rn,Lewis number Le and modified diffusivity ratio N_A respectively in the case of free-free isothermal and iso-nano concentration boundaries for different non-uniform temperature profiles.

From the figure (5.2), it is observed that as the value of concentration Rayleigh number Rn increase,Ra also increase and hence stabilizes the system indicating delay in the convection onset. Positive values of Rn are taken which indicates that density of particle decreases upwards.The reason for the stabilizing effect of Rn is that because of the increase in nanoparticle concentration and difference in temperature between the plates,energy gets transfered among fluid and nanoparticles,which in turn delays the convection.

The effect of Lewis number Le on the stability of the system is given by figure (5.3).The large values of Le are chosen in this problem because Le is inversely proportional to brownian diffusion coefficient D_B which takes small values.Therefore increase in Le,ie. the decrease in the brownian diffusion coefficient leads to increase in Ra hence stabilizing the system.

The effect of modified diffusivity ratio N_A on system stability is depicted in figure(5.4). From the figure we observe that increase in N_A decreases the value of Ra. The reason is that since N_A is directly proportional to thermophoresis diffusion coefficient D_T,whenever N_A increase,D_T also increses and reduces Ra. Hence the increment in modified diffusivity ratio leads to system destabilization.

Figures (5.5-5.7) and (5.8-5.10) respectively are the plots for Rayleigh number Ra versus various parameters like concentration Rayleigh number Rn,Lewis number Le and modified diffusivity ratio N_A in the cases of rigid-rigid and rigid-free isothermal and iso-nano concentration boundaries for different non-uniform temperature profiles.We observe that the results in these cases are quantitatively similar to that of free-free isothermal and iso-nano concentration boundary.

From the figures it is observed that, $R_{a_c}^{FF} < R_{a_c}^{RF} < R_{a_c}^{RR}$, where the superscripts denotes the various boundary combinations.

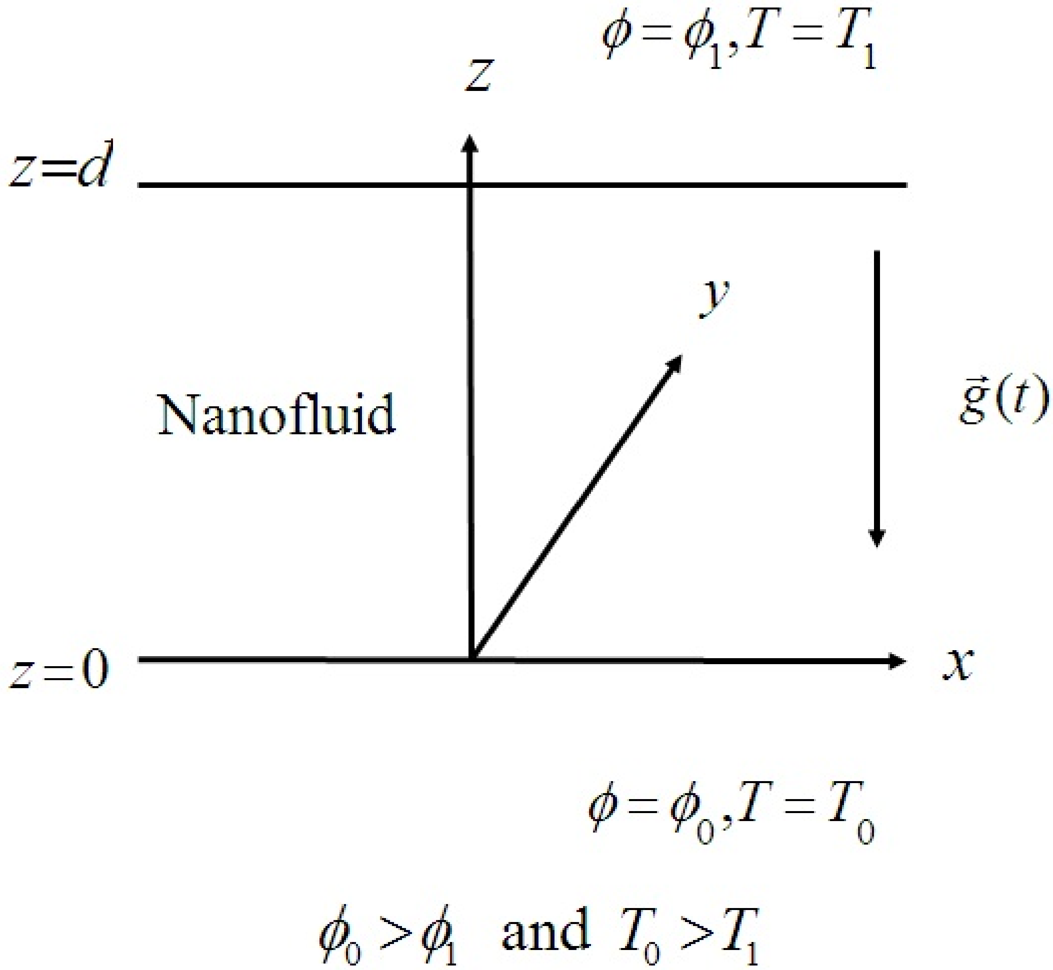

Figure 5.1: Physical Configuration

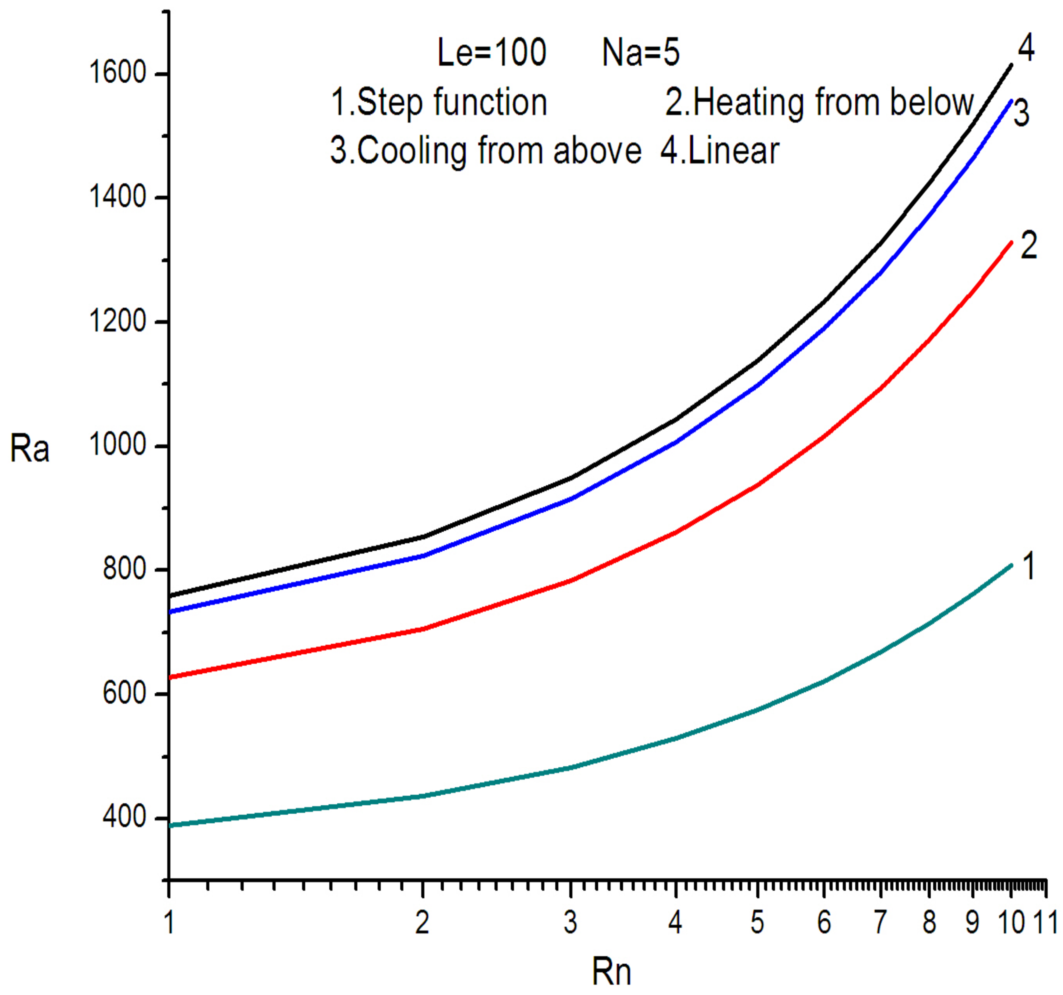

Figure 5.2: Plot of concentration Rayleigh number Rn versus Rayleigh number Ra for different temperature profiles for free free isothermal boundary.

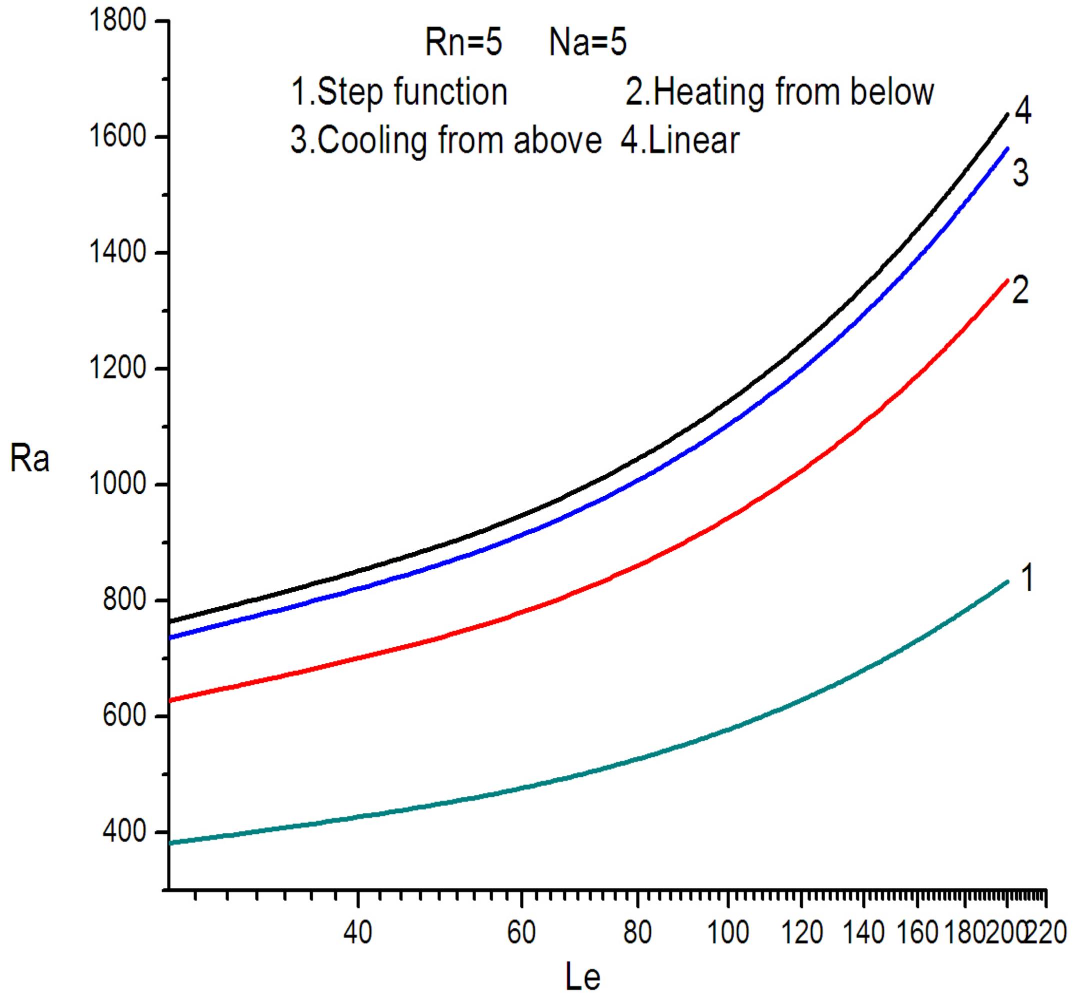

Figure 5.3: Plot of Lewis number Le versus Rayleigh number Ra for different temperature profiles for free free isothermal boundary.

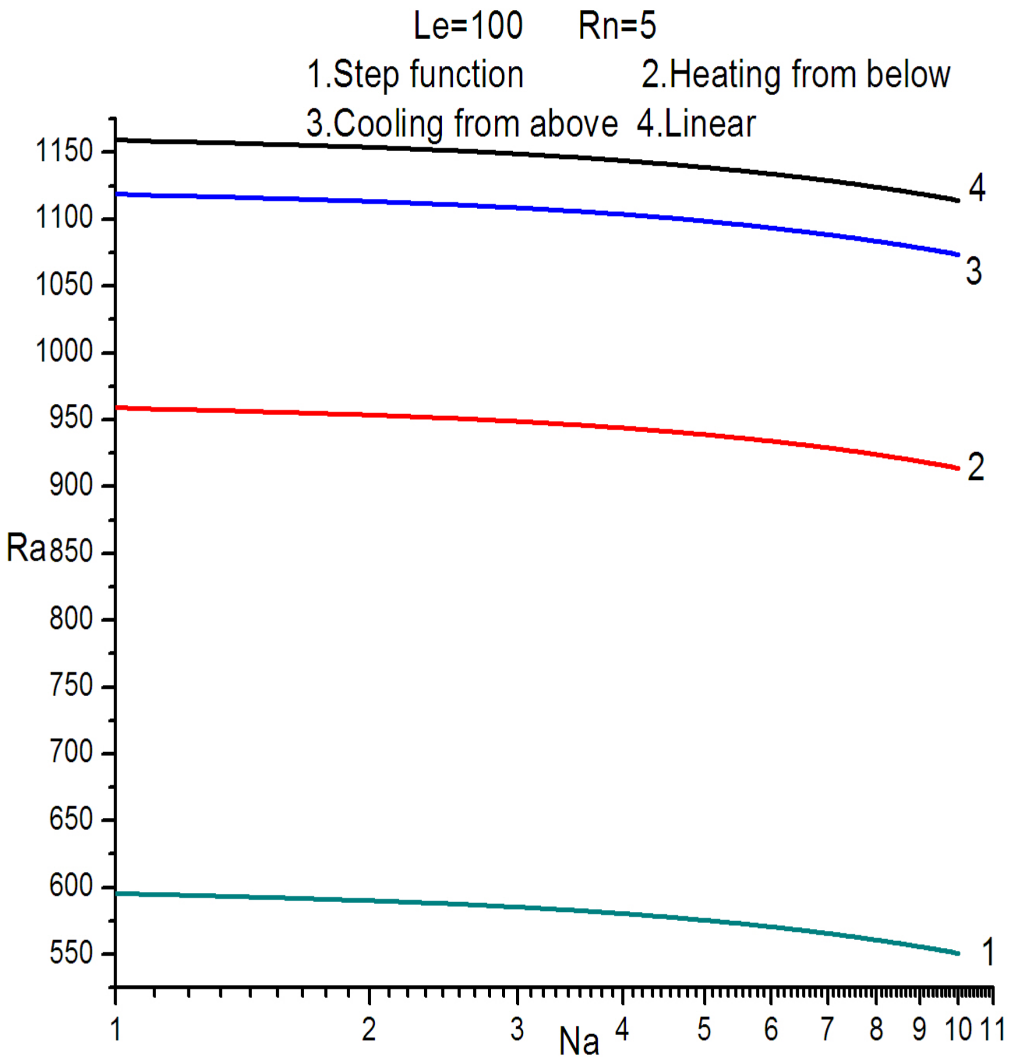

Figure 5.4: Plot of modified diffusivity ratio N_A versus Rayleigh number Ra for different temperature profiles for free free isothermal boundary.

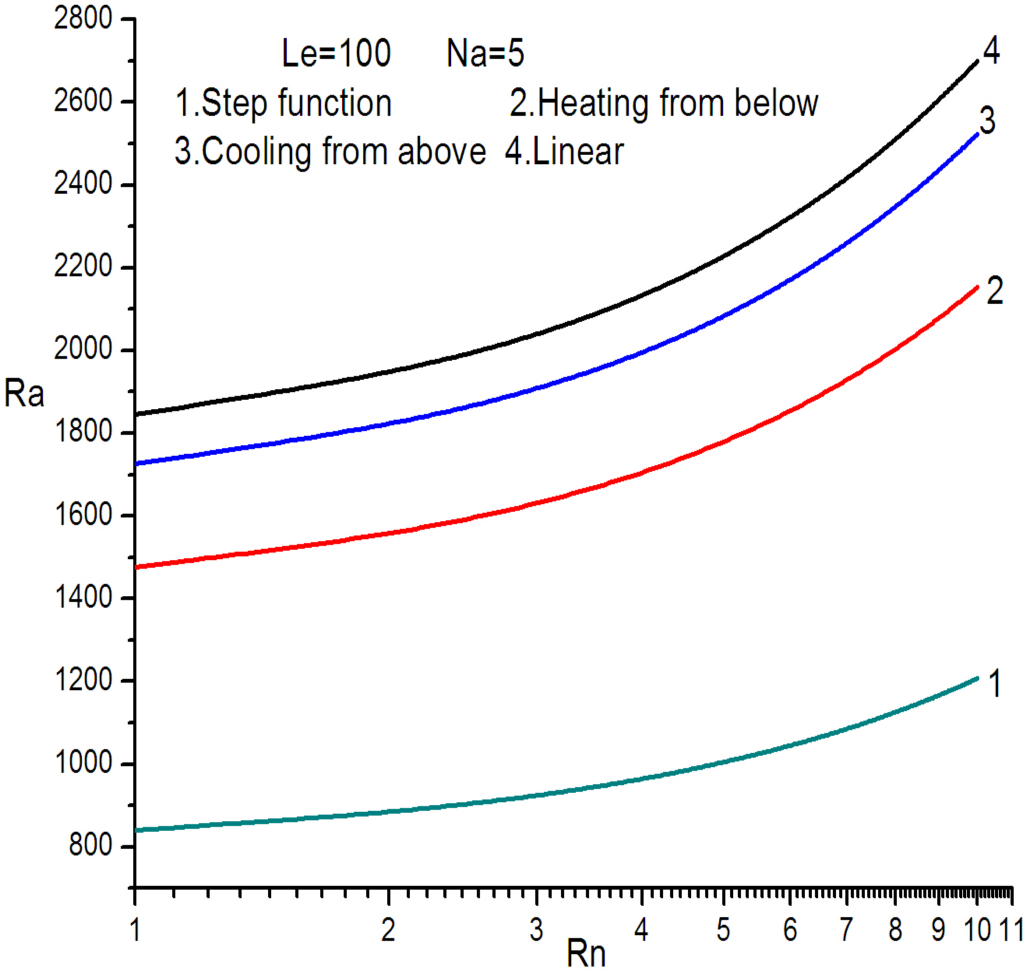

Figure 5.5: Plot of concentration Rayleigh number Rn versus Rayleigh number Ra for different temperature profiles for rigid rigid isothermal boundary.

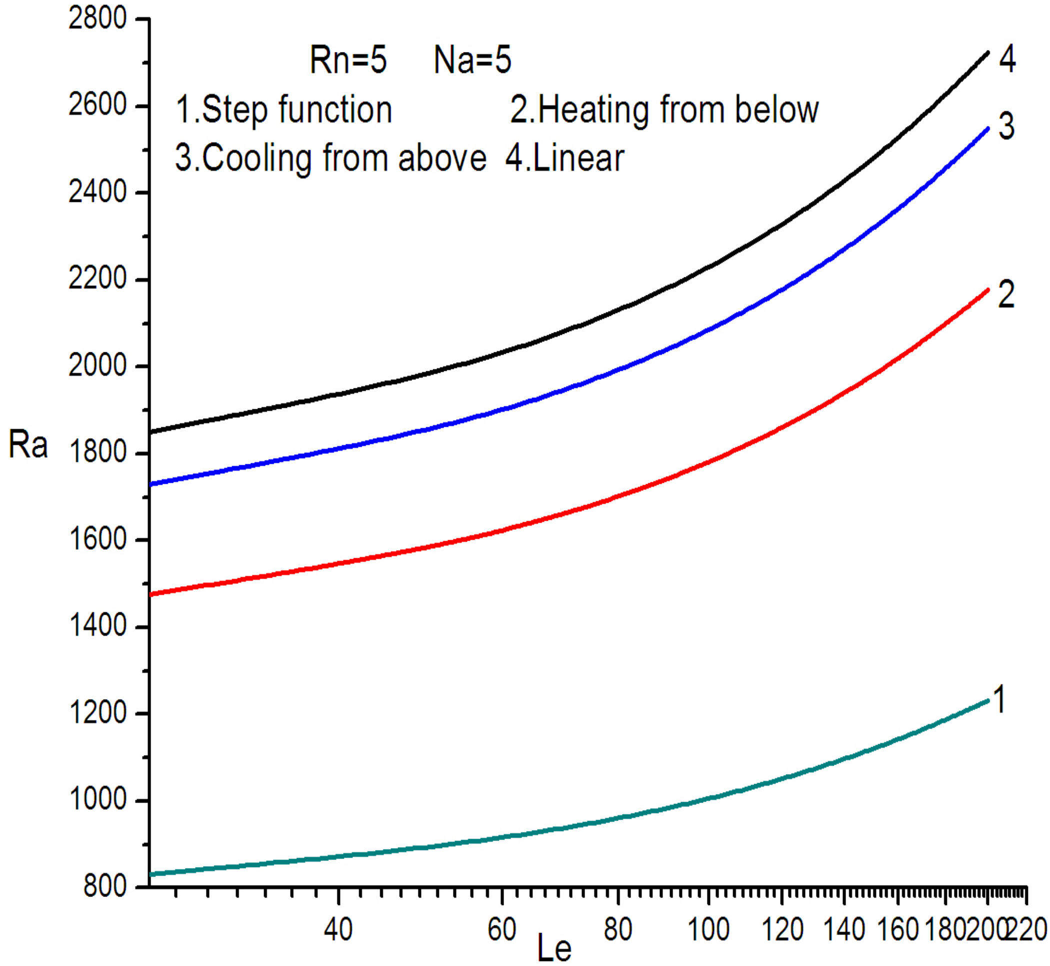

Figure 5.6: Plot of Lewis number Le versus Rayleigh number Ra for different temperature profiles for rigid rigid isothermal boundary.

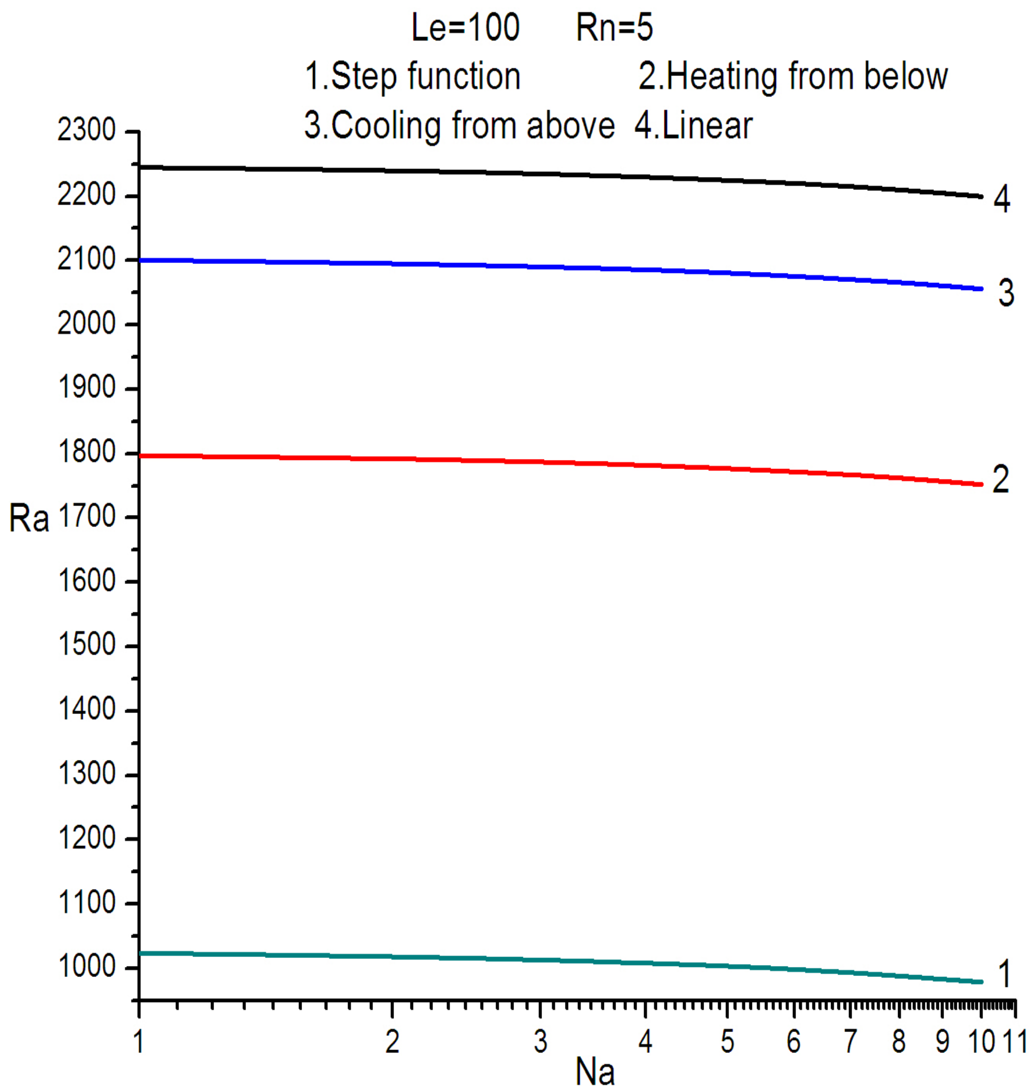

Figure 5.7: Plot of modified diffusivity ratio N_A versus Rayleigh number Ra for different temperature profiles for rigid rigid isothermal boundary.

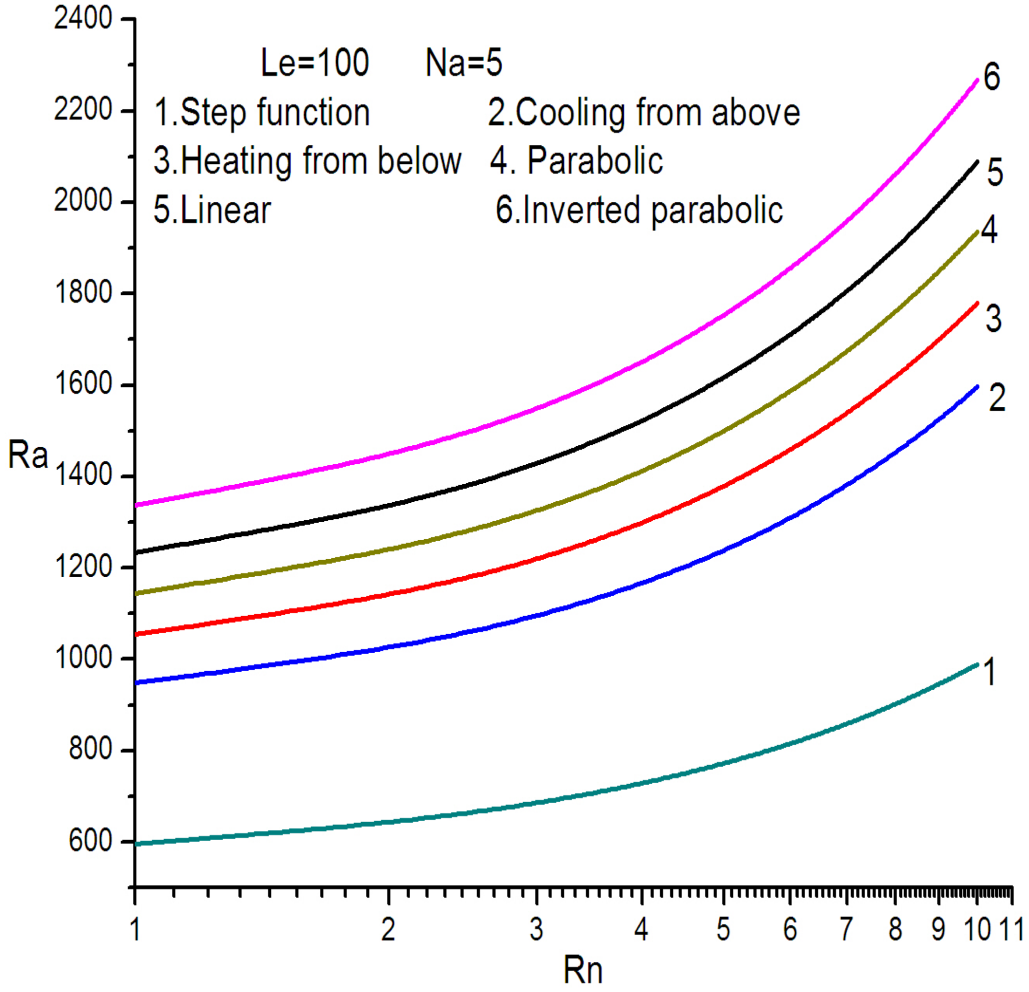

Figure 5.8: Plot of concentration Rayleigh number Rn versus Rayleigh number Ra for different temperature profiles for rigid-free isothermal boundary.

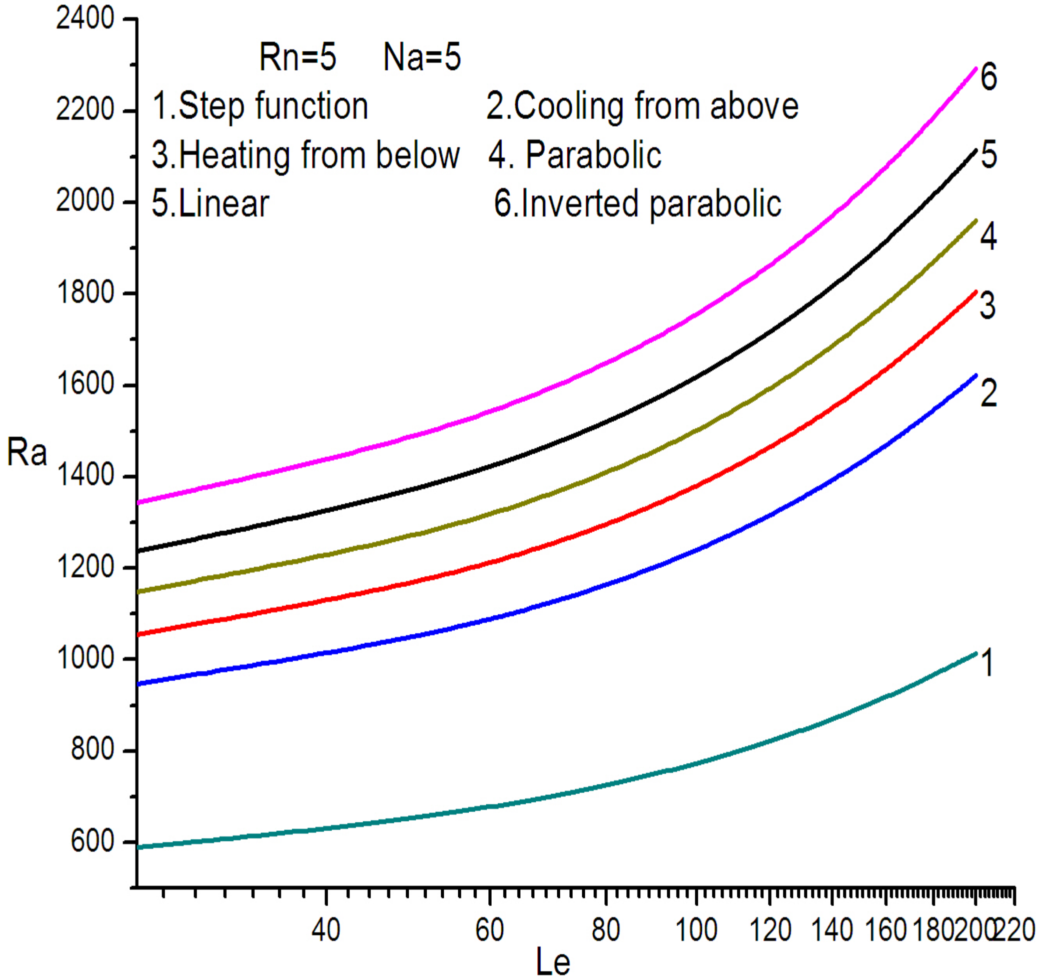

Figure 5.9: Plot of Lewis number Le versus Rayleigh number Ra for different temperature profiles for rigid-free isothermal boundary.

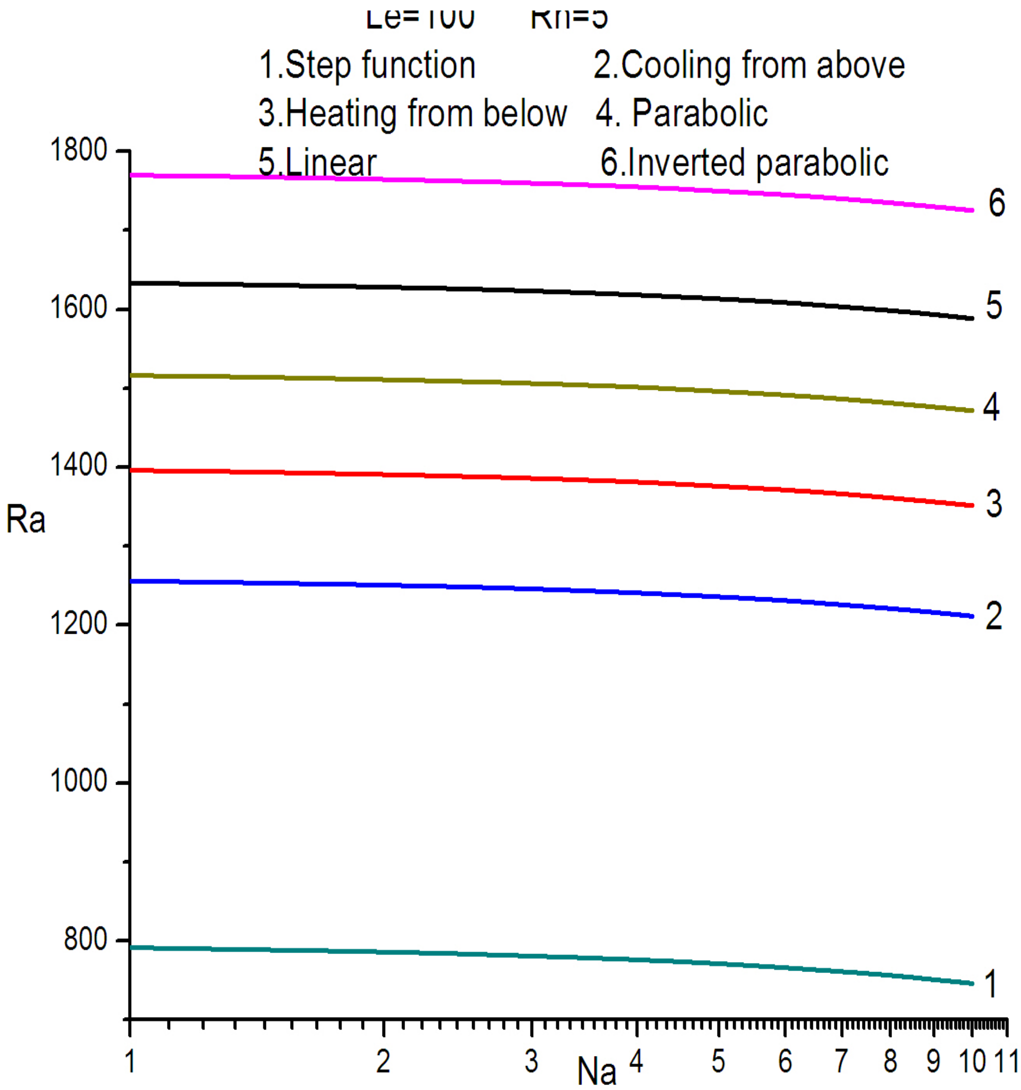

Figure 5.10: Plot of modified diffusivity ratio N_A versus Rayleigh number Ra for different temperature profiles for rigid-free isothermal boundary.

Chapter 6

Summary and Conclusions

In this research work, the effect of the non-uniform temperature profiles on the onset of Rayleigh-B$\acute{e}$nard convection in a nanofluid is investigated.The bottom heavy distribution of nanoparticles are considered.Linear theory based on normal mode analysis is considered in the study.The expression for Rayleigh number in terms of R_n,L_e,N_A and $f\left(z\right)$ are obtained using Galerkin method.The following are the conclusion drawn from the study.

1. In the case of nanofluids more amount of heat is required compared to Newtonian fluid.
2. Step function and linear profile are the most destabilizing and stabilizing temperature gradients in the case of symmetric boundaries.
3. Step function and inverted parabolic are the most destabilizing and stabilizing temperature gradients in the case of non-symmetric boundaries.
4. Concentration Rayleigh number R_n and Lewis number L_e stabilizes the system.
5. Modified diffusivity ratio N_A destabilizes the system.
6. Onset of convection can be controlled by using the appropriate non-uniform temperature profile.
7. As a general result,we observe that R(Nano fluid)$>$R(Newtonian fluid) and $R_{a_c}^{FF} < R_{a_c}^{RF} < R_{a_c}^{RR}$.

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