The Effect of Temperature Modulation on the Onset of Rayleigh-Bénard Convection in a Dielectric Couple Stress Fluid with Maxwell-Cattaneo Law

A Dissertation Submitted in Partial Fulfillment of the Requirements for the Award of the Degree of

Master of Philosophy

in

Mathematics

by

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Approval of Dissertation

Dissertation entitled **The Effect of Temperature Modulation on the Onset of Rayleigh-Bénard Convection in a Dielectric Couple Stress Fluid with Maxwell-Cattaneo Law** by Milan Maria Mathew, Reg. No. 1435303 is approved for the award of the degree of Master of Philosophy in Mathematics.

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DECLARATION

I, Milan Maria Mathew, hereby declare that the dissertation, titled **The Effect of Temperature Modulation on the Onset of Rayleigh-Bénard Convection in a Dielectric Couple Stress Fluid with Maxwell-Cattaneo Law** is a record of original research work undertaken by me for the award of the degree of Master of Philosophy in Mathematics. I have completed this study under the supervision of **Dr S. Pranesh**, Professor, Department of Mathematics.

I also declare that this dissertation has not been submitted for the award of any degree, diploma, associateship, fellowship or other title. It has not been sent for any publication or presentation purpose. I hereby confirm the originality of the work and that there is no plagiarism in any part of the dissertation.

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CERTIFICATE

This is to certify that the dissertation submitted by Milan Maria Mathew (Reg. No. 1435303) titled **'The Effect of Temperature Modulation on the Onset of Rayleigh-Bénard Convection in a Dielectric Couple Stress Fluid with Maxwell-Cattaneo Law'** is a record of research work done by her during the academic year 2014 - 2016 under my supervision in partial fulfillment for the award of Master of Philosophy in Mathematics.

This dissertation has not been submitted for the award of any degree, diploma, associateship, fellowship or other title. It has not been sent for any publication or presentation purpose. I hereby confirm the originality of the work and that there is no plagiarism in any part of the dissertation.

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Contents

Abstract

Rayleigh-B$\acute{e}$nard Convection in a couple stress fluid with electric field is considered in this research work and the impact of thermal modulation on the onset of convection is studied. Maxwell-Cattaneo Law is substituted for the traditional Fourier Law. Accordingly, the high speed for heat exchange which is a consequence of the traditional law is avoided. The Venezian method is used to arrive at the critical Rayleigh number, wave number and correction Rayleigh number. The problem is analysed under three different instances of oscillating temperature field: (a) Symmetric with $\varphi = 0$, (b) asymmetric with $\varphi = \pi$, (c) modulation of temperature of bottom wall with $\varphi = -i\infty$. It is observed that sub-critical movement happens due to temperature modulation. It is also found that temperature modulation can advance or postpone the onset of convection.

Chapter 1

Introduction

1.1 Objective and Scope of the Dissertation

Conduction, convection and radiation are the three modes of exchange of heat. Convection is of two types: natural convection and forced convection. Natural Convection happens due to the migration of fluid particles which occurs due to the difference in density of the fluid. Rayleigh-Bénard Convection, a kind of convection that takes place naturally in a plane fluid layer results in the evolution of a special arrangement of convection cells known as Bénard cells.

A stratum of fluid is taken between two parallel plates. Initially the temperature of the bottom plate and upper plate remains the same and the system is in a state of equilibrium. The temperature of the bottom plate is slightly increased and the heat gets transferred to the fluid by conduction. Thus there is a temperature difference between both the plates. But this temperature gradient is not strong enough to overcome the thermal conductivity and viscosity of the fluid and hence there is no movement of particles. But at a certain point of time the temperature gradient becomes strong enough to overcome the viscous damping force of the fluid and thus the convection starts. The density of the lower fluid layer decreases and the lighter fluid rises up. But it looses its heat when it reaches at the top. Hence the gravity tries to pull the cooler denser fluid from top to bottom. Thus there is a continuous movement of fluid particles which results in the formation of convection cells. Thus the system is in a state of instability.

A dimensionless parameter called Rayleigh number is used to express the balance between the gravitational force and the viscous damping force which is given by:

$$R = \frac{\alpha g \rho_0 d^3 \Delta T}{\mu \kappa}$$

where, α is the coefficient of thermal expansion, g is acceleration due to gravity, d is the distance between the plates, ΔT is the difference in temperature, μ is co-efficient of viscosity and κ is thermal conductivity.

Couple Stress fluid is a critical class of non-Newtonian fluids. It is alluring to study this type of fluid with suspended particle due to its increasing significance and applications in the modern technology. Stokes developed the couple stress fluid theory in 1966. The possession of greater viscosity and the existence of couple stresses together with the classical Cauchy stress is an important property of such fluids. Couple stresses arises as a result of the mechanical interactions in a fluid. Polar effects is a distinct feature of this type of fluid. The principle highlight of couple stress fluid is that it introduces an effect which is independent of size which is not considered in the classical continuum mechanics. The investigation of the lubrication of synovial joins in human body is an important application of couple stress fluid. It has many other applications such as in a bath where the metallic plates are cooled, solidifying the liquid crystals and so forth.

The Fourier law of heat conduction states that the fluctuation of heat inside of a medium varies proportionally with the temperature gradient that occurs locally in the system. But this law concludes that the heat propagates with a high speed, which is an unphysical result. This motivated many researchers like Cattaneo[11], Lindsay and Straughan[21], Straughan and Franchi[57], Puri and Kythe[41], Puri and Jordan [40], Siddheshwar[49], Pranesh and Kiran[36], Pranesh and Smita[39] to adopt a non traditional law known as Maxwell-Cattaneo law to study Rayleigh-B$\acute{e}$nard Convection.

Maxwell-Cattaneo equation is given by:

$$\tau \frac{d\vec{Q}}{dt} + \vec{Q} = -\kappa \bigtriangledown T$$

This equation together with energy equation forms a hyperbolic equation which defines that heat propagates in a finite speed.

The idea of maintaining a temperature gradient across the boundaries is an effective method of controlling convection. Temperature gradient is dependent on both space and time in most of the practical applications. The transient warming or cooling at the limits have led to the origin of non uniform temperature gradient and subsequently the temperature gradient relies upon both position and time. This type of problem named as the thermal modulation problem includes a temperature profile that relies upon both position and time and can be utilized to control the convection by legitimate alterations of amplitude and frequency of modulation. Venezian[64] first explored on the impact of such problems on the thermal instability. He found that an appropriate regulation of the boundary temperatures can lead to the advancement or delaying of convection. Later researchers like Siddheshwar and Pranesh[53], Bhadauria[2], Malashetty and Basavaraja[23], Pranesh and Kiran[38], Takashima and Ghosh[60] and so forth have explored thermal modulation problems.

A strong electric field can induce bulk motions in a system. Electro convection is gaining importance since it replaces large switching circuits which consumes space in machines. It is used in designing more efficient heat exchangers in jet engines. Electrohydrodynamics were studied by many researchers like Ezzat and Othman[14], Takashima and Ghosh[14].

The main aim of the study is to investigate on the influence of temperature modulation on the onset of Rayleigh-Bénard convection in a couple stress fluid in the presence of electric field with Maxwell-Cattaneo law.

Chapter 2

Literature Review

The literature relevant to the research topic is reviewed and is given below. Thomson[61] reported the earliest study done on thermal instability. Later a wider description of the convective flow was given by Bénard[1]. It was Lord Rayleigh[42] who successfully analysed that the convection occured due to the temperature gradient and hence known as Rayleigh Bénard Convection. This theory was extended to various boundaries by Jeffrey [17] [16]. However the most complete theory related to the problem was reported by Pellow and Southwell[32]. A very wide study of the theoretical and experimental investigations was given by Ostrach[30].

Sparrow *et al.*[58] and Roberts[43] considered the onset of convection which incorporates a temperature profile which is parabolic in nature. The impact of non uniform temperature profile in a saturated permeable medium was considered by Nield[29]. The formation of hexagonal cells and sub critical instabilities were discussed by Ruby[44]. Investigations on Magneto-convection was carried out by Fermi[15]. The steady linear and non-linear Magneto-convection in the three-dimensional situation was analysed by Rudraiah[45]. In case of porous medium, the effect of suction and injection on outbreak of Rayleigh-Bénard convection was studied by Siddheshwar[50].

The effect of non-linear temperature gradient on Marangoni convection in oscillatory and stationary states was investigated by Chiang[12], whereas Idris *et al.* investigated the same for micropolar fluid. A thermal non-equilibrium model was utilized by Malashetty and Sridhar[25] to investigate the instability in case of convection in a porous layer saturated with Maxwell fluid.

The literature review pertaining to Rayleigh-Bénard convection in couple stress fluids are reviewed as follows.

The couple stress fluid theory was introduced by Stokes[56]. Couple stress fluid theory is a wider representation of the viscous Newtonian fluids that permits the existence of couple stresses in the fluid. The idea of couple stress is an after effect of the mechanical interactions within a fluid medium. There exists a complicated relationship between shear stress and flow field in a couple stress fluid. The problem of peristaltic flow of couple stress fluid with zero

Reynold's number and estimation of wavelength was studied by Srivastava[55]. Under given conditions he found that, when compared to a Newtonian fluid, the decrease in pressure is more prominent in a couple stress fluid. The increase in couple stress parameter increments the rise in pressure.

Sharma and Thakur[47] analysed a stratum of couple stress fluid which was electrically conducting in penetrable medium in the presence of magnetic field. They found that presence of couple stress and magnetic field delays the convection while the penetrability of the medium hastens it in stationary convection. They found that oscillatory modes of convection are introduced in the system by magnetic field.

A layer of couple stress fluid in permeable medium which is conducting and heated from below was considered by Sharma and Shivani[48] in the presence of a uniform magnetic field. They found that the convection is deferred by couple stress parameter in stationary convection.

Siddheshwar and Pranesh[54] investigated the influence of Rayleigh-Bénard convection in Bous sinesq Stokes suspension in linear and non-linear analysis. A normal mode solution was used in linear analysis and a double fourier series representation for the analysis in non-linear case. The effects of suspended particles on convection is studied by comparing with a clean fluid.

A stratum of couple stress fluid which was heated and soluted from below in a porous medium was studied by Sunil *et al.*[59]. Suspended particles in the fluid was also considered. In stationary convection, they found that the convection is stabilized by couple stress parameter.

Sharma and Mehta[46] considered the layer of compressible rotating couple stress fluid which is soluted from below. In the case of stationary convection, they found that the influence of the viscosity of couple stress fluid on convection depends on the rotation parameter.

Malashetty *et al.*[27] studied both linear and non-linear stability analysis to study the double diffusive convection with Soret effects in a couple stress fluid. Linear analysis was carried out by solution in normal mode and non-linear analysis by the two fold fourier series representation.They found that the Soret effect stabilizes the system.

Ezzat *et al.*[13] investigated the effect of magnetic field using boundary layer equations. This model was used to investigate the effect of free convection streams on the transport of polar fluid through a penetrable medium which was restricted by plane vertical surfaces.

The effects of dust particles, coriolis force and magnetic field in a couple stress fluid was analysed by Vivek and Sudhir[19]. In case of stationary convection, coriolis force was found to stabilize the system while dust particles destabilize the system. Depending on range of parameters, they observed that the couple stress and magnetic field can make the system stable or non stable. They also found that the oscillatory modes were introduced in the system due to the influence of magnetic filed and coriolis force.

The peristaltic outflow of a couple stress fluid in porous medium with magnetic field was analysed by Pande and Chaube[31]. Results obtained have demonstrated that the mean speed of the flow at the limits can be diminished with rise in couple stress parameter.

The significance of time dependent temperature at the boundaries on the onset of convection in an electrically conducting couple stress fluid was investigated by Pranesh and Sangeetha[35]. It was found that convection can be controlled by applying the electric field.

Now we have the literature review pertaining to Rayleigh Bénard Convection using Maxwell Cattaneo Law.

The Fourier Law of Heat Conduction states that the heat flow inside a system varies proportionally with the temperature gradient in the system. This law concludes that heat propagates with infinite velocity. In order to get rid of this unphysical result, Maxwell[28] and Cattaneo[11] adopted a non classical law in studying the Rayleigh Bénard Convection. Lindsay and Straughan[21], Straughan and Franchi[57] also adopted this heat flux model to study the convection wherein they allowed the thermal waves to be of finite speed.

Lebon and Cloot[20] studied Bénard-Marangoni problem by substituting the Fourier Law by Maxwell Cattaneo Law. They found that only oscillatory convection is possible when buoyancy is the single factor of instability. The also studied the consequences when Jaumann derivative is substituted by extending the work of [57].

Puri and Kythe[41] investigated heat conduction effects in Stokes problem using Maxwell-Cattaneo Fox model. Maxwell-Cattaneo Fox model was used by Puri and Jordan[40] to study the wave structure in Stokes second problem in a dipolar fluid. The effect of thickness of the film and skin friction are also studied.

Siddheshwar[49] studied Rayleigh Bénard Convection in a ferromagnetic fluid of second order by using Maxwell-Cattaneo law. He found that for heating from above, oscillatory convection is possible and the Cattaneo number influences the critical Rayleigh number.

Pranesh and Kiran[36] investigated Rayleigh Bénard Magneto Convection in a micropolar fluid which is electrically conducting where they substituted Maxwell-Cattaneo Law in place of

Fourier law. The classical Fourier law assumes that the heat propagation is of infinite speed. They found that the non classical law assumes wave type transport of heat and hence the unphysical results are avoided.

Rayleigh-Bénard convection in a second order fluid was investigated by Pranesh and Smita[39] using Galerkin technique with Maxwell-Cattaneo law. The found that the critical eigen values of the problem is less than that of classical result and over stability is the preferred mode of convection.

The influence of suction-injection on the onset of Magneto-Convection in a micropolar fluid was studied by Pranesh and Kiran[37] with the non classical law employing Rayleigh-Ritz technique. They found that pro gravity suction injection combination stabilizes the system and anti gravity suction-injection combination destabilizes the system.

The literature review related to temperature modulation is discussed in the following paragraphs.

Venezian was the first to consider the effect of time dependent temperature in a fluid layer heated from below. He obtained the solution for small amplitude and found that for small frequency the system becomes unstable.

Siddheshwar and Pranesh[52] investigated the influence of thermal and gravity modulation on the onset of magneto-convection in fluids which are weakly conducting with internal angular momentum. Venezian approach was used to find the critical Rayleigh number. They observed that modulation of the boundary temperatures can cause sub-critical movement and the system is stabilized by gravity modulation.

Siddheshwar and Pranesh[53] repeated the previous study for electrically conducting fluid and the results were presented against the background of the previous work. It was found that the magnitude of the eigen value is less than that of the previous case.

Malashetty and Basavaraja[23] investigated the influence of temperature/gravity modulation which is time dependent on the onset of convection in an anisotropic permeable medium filled with Boussenesq fluid by conducting a linear stability analysis. The technique of perturbation was used to arrive at the critical Rayleigh number and wave numbers for modulations of slight amplitude. The change in critical Rayleigh number was expressed in terms of viscosity ratio, porous parameter, anisotropy parameter and the frequency of the modulation. They found that the thermal modulation can lead to a stable or instable system while modulation of gravity progresses the convection. It was also found that the stability of the system can be influenced by a small anisotropy parameter.

Bhadauria[2] employed Floquet theory to study the layer of fluid considered between two rigid boundaries. Only infinitesimal disturbances were considered and the temperature distribution was comprised of a consistent part and a time dependent oscillatory part.He observed that the perturbations are either in sync with the temperature field or is with frequency that is half of the temperature field.Badauria[3] studied the layer of fluid between two inflexible boundaries. A steady part and an oscillating part was considered for the temperature distribution and very small disturbances were considered. Critical Rayleigh number was computed numerically for various estimations of Prandtl number and frequency of modulation.

The stability of a stratum of fluid which was heated from below and cooled from above was investigated by Bhadauria and Lokenath[9]. Not withstanding the consistent temperature distinction between the boundaries, a perturbation depending on time was additionally applied. For various estimations of Prandtl number and frequency, the critical Rayleigh number was figured.

Malashetty and Basavaraja[24] studied the influence of thermal modulation on double diffusive convection in an anisotropic porous medium saturated by the fluid.A linear stability analysis was employed for the study. Critical Rayleigh number and wave number was obtained by using the technique of perturbation. The modulation frequency, viscosity ratio, anisotropy parameter, porous parameter, Prandtl number, ratio of diffusivity and solute Rayleigh number was used to model the correction thermal Rayleigh number. It was concluded that the modulation of wall temperatures can progress or postpone the start of double diffusive convection.

Bhadauria[4] studied the instability in an fluid which is electrically conducting in the presence of a vertical magnetic field. Both the boundaries were modulated for the distribution of the temperature. Floquet theory was employed in analysing the integrated effect of modulation and magnetic field. Magnetic field was found to lead to a stabilized system and the modulation could influence the convection by legitimate adjustments of frequency.

Malashetty and Swamy[8] investigated the stability of a fluid in porous medium which is rotating with thermal modulation. The effect of infinitesimal perturbation was analysed using linear stability analysis. Darcy-Rayleigh number and wave number was computed using the perturbation technique. The change in critical Darcy-Rayleigh number was computed as function of Darcy-Prandtl number, frequency and Taylor number. It was found that convection is hastened when the wall temperature is modulated symmetrically and when the bottom wall is modulated but deferred when the modulated of temperature is asymmetrical.

The consolidated impact of thermal modulation and magnetic field on the commencement of convection in a porous medium drenched with electrically conducting fluid was examined by Bhadauria[7]. Correction Rayleigh number is obtained in terms of Darcy Chandrasekhar number, frequency of modulation, magnetic Prandtl number and Darcy number. He found that the magnetic field delays the convection in the system and proper adjustments of frequency of modulation leads to advancing or delaying the onset of convection.

A stratum of fluid heated from below was examined by Bhadauria *et al.*[8]. The temperature gradient comprises of a constant part and an oscillating part. They found that the modulation near the critical Rayleigh number generated a range of hexagons which were stable.

The thermal instability in a fluid which is electrically conducting was studied by Siddheshwar and Abraham[51] with boundaries are subjected to time periodic temperatures. They found that when the temperature modulation was asynchronous, the system is most stable.

Bhadauria and Srivastava[10] studied the instability in a porous medium which was saturated with electrically conducting fluid with the boundaries modulated and in the presence of a vertical magnetic field. Critical Darcy Rayleigh number was calculated using perturbation method. The fluid parameters were found to have stabilizing or destabilizing effects and hence convection can be advanced or delayed.

The impact of thermal and gravity modulation in an anisotropic permeable medium was studied by Vanishree[63] by employing a linear stability analysis. Perturbation technique was used to compute the correction Rayleigh number. This problem illustrated a method of regulating convection.

Pranesh[33] using linear stability analysis, studied the impact of electric field and boundary temperature which is dependent on time in a micropolar fluid. The correction Rayleigh number was obtained by the regular perturbation technique and eigen values were obtained using Venezian approach. Temperature field were examined in three cases. He found that the system is extremely stable when the wall temperatures are modulated asymmetrically.

Pranesh and Sangeetha[35] investigated on the influence of thermal modulation on Rayleigh-Bénard convection in an electrically conducting couple stress fluid using linear stability analysis. Regular perturbation method was adopted in the computation of correction Rayleigh number and eigen values were obtained using Venezian approach. Three instances of temperature field were examined. They noted that the system is of utmost stability when the temperature of the walls are modulated asymmetrically.

Pranesh and Kiran[38] studied the effect of thermal modulation and magnetic field on convection where the non classical law was employed. The critical Rayleigh number, correction Rayleigh number and wave number are computed using Venezian approach. Three cases of oscillating temperatures are considered. It was found that the system is most stable in out of phase modulation.

Next we will look into the literature review regarding convection in dielectric fluids.

Takashima and Ghosh[60] investigated electrohydrodynamic instability in a viscoelastic fluid. It was observed that the fluid layer with thickness not exactly around 0.5 mm give rise to oscillatory modes of instability and in this case buoyancy demands more importance than the force of electrical origin.

Ezzat and Othman[14] analysed the impact of vertical electric field on the thermal instability in a rotating micropolar fluid. The eigenvalue equation was obtained by the power series method.

Siddheshwar gave an analogy between the electro hydrodynamic and ferro hydrodynamic instability in Newtonian fluids. It was found that the Rayleigh Bénard problem in dielectric liquids can be obtained from an analogous problem in ferromagnetic liquids.

Rayleigh-Bénard convection in a liquid which is electrically conducting with time dependent temperature was investigated by Siddheshwar and Abraham[51]. It was found that the Prandtl number and dielectric parameters has opposing effect at high frequencies.

The effect of electric field and and non uniform temperature difference were studied by Pranesh and Riya[34]. They observed that there is a possibility of controlling the instability in convection in a micropolar fluid by proper adjustments in the electric field.

Pranesh[63] studied the influence of imposed non uniform boundary temperature and electric field in a micropolar fluid. The study gave an exterior means to control the convection in the internal state with electric field.

Investigations were carried out by Joseph *et al.*[18] to study the effects of electric field and non-uniform temperature gradient on the onset of Rayleigh-Bénard-Marangoni convection in a micropolar fluid using Galerkin technique. The effect of electric Rayleigh number on the onset of convection was investigated. They found that convection in micropolar fluid can be effectively controlled by an electric field which is applied externally.

Chapter 3

Basic Equations, Boundary Conditions, Approximations, Scaling and Dimensionless Parameters

The governing equations, assumptions, boundary conditions, scaling and the dimensionless parameters considered in this problem are discussed in this chapter.

3.1 Basic Equations

Conservation of mass:
The equation of continuity is given generally by

$$\frac{\partial \rho}{\partial t} + \bigtriangledown . (\rho \vec{g}) = 0. \tag{3.1}$$

For a fluid which is incompressible, ρ is constant

$$\Rightarrow \bigtriangledown . \vec{q} = 0. \tag{3.2}$$

Conservation of Linear Momentum:

$$\rho_0 \left(\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\bigtriangledown)\vec{q} \right) = - \bigtriangledown p + \mu \bigtriangledown^2 \vec{q} - \acute{\mu} \bigtriangledown^4 \vec{q} + \rho \vec{g} + (\vec{P}.\bigtriangledown)\vec{E}. \tag{3.3}$$

Conservation of Energy:

$$\frac{\partial T}{\partial t} + (\vec{q}.\bigtriangledown)T = - \bigtriangledown . \vec{Q}. \tag{3.4}$$

Maxwell Cattaneo Heat Flux Equation:

$$\tau \left[\vec{Q} + \vec{\omega}_1 \times \vec{Q} \right] = -\vec{Q} - \kappa \bigtriangledown T. \tag{3.5}$$

Equation of state:

This equation is obtained from the Taylor series expansion of fluid density considered at the reference temperature T_0 where the higher order terms are eliminated.

$$\rho = \rho_0 \left(1 - \alpha \left(T - T_0\right)\right). \tag{3.6}$$

Electrical Equation:

$$\nabla \times \vec{E} = 0, \tag{3.7}$$

$$\nabla \cdot \left(\epsilon_0 \vec{E} + \vec{P}\right) = 0,\ \vec{P} = \epsilon_0 \left(\epsilon_r - 1\right)\ \vec{E}. \tag{3.8}$$

Equation of state for dielectric constant:

$$\epsilon_r = \left(1 + \chi_e\right) - e\left(T - T_0\right). \tag{3.9}$$

3.2 Approximations

The following assumptions are made in this dissertation.

1. The fluid considered is homogeneous and incompressible.
2. The validity of Boussenesq approximation and hence the equation of continuity becomes

$$\nabla \cdot \vec{q} = 0.$$

3. The gravity acts vertically downwards.
4. The fluid parameters namely thermal diffusivity, viscosity are all assumed to be constant.
5. The dielectric constant ϵ_r can be linearly expressed in terms of temperature.

3.3 Boundary Conditions

3.3.1 Boundary Conditions on Velocity

Relying upon the type of boundary surface whether they are free or rigid, the no slip conditions, Cauchy's Stress Principle and mass balance lead to the formation of boundary conditions on velocity. Here in this problem a free surface with no surface tension is considered.

Therefore the velocity boundary conditions are

$$w = \frac{\partial^2 w}{\partial z^2} = 0. \tag{3.10}$$

3.3.2 Boundary Conditions on Temperature

The limit conditions on temperature are computed by considering the heat conducting property of the boundaries

Fixed Surface Boundary

In case of high thermal conductivity and large thermal capacity,the temperature is time independent and uniform.Therefore at the boundaries,

$$T = 0 \tag{3.11}$$

This condition is called isothermal boundary conditions

Temperature Modulation Boundary Conditions

The following conditions are considered at the boundaries since the temperature profile depends on position and time.

$$T\left(0,t\right) = T_0 + \frac{1}{2}\Delta T\left[1 + \epsilon\cos\omega t\right], \tag{3.12}$$

$$T\left(d,t\right) = T_0 - \frac{1}{2}\Delta T\left[1 - \epsilon\cos\left(\omega t + \varphi\right)\right]. \tag{3.13}$$

3.3.3 Boundary Conditions on Electric Potential

The boundary conditions on electric potential is given by

$$D\phi = 0 \; at \; z = 0 \; and \; z = 1. \tag{3.14}$$

3.4 Scales Used for Non-Dimensionalisation

In order to understand the relative importance of each term used, we make the equations dimensionless by the introduction of characteristic quantities. The scales used for non dimensionaliation in this problem are as follows:

Quantity	Characteristic quantity used for scaling
Time	$\frac{d^2}{\kappa}$
Length	d
Temperature	ΔT
Electric Potential	$\frac{eE_0 d\Delta T}{1+\chi_e}$

3.5 Dimensionless Parameters:

The following are the dimensionless parameters that appear in this problem:

1. **Rayleigh number:**
   Rayleigh number is given as:
   $$R = \frac{\rho_0 \alpha g d^3 \Delta T}{\mu \kappa}.$$
   Rayleigh Number which was named after Lord Rayleigh gives the relation between the buoyancy and viscosity of a fluid. Rayleigh number greater than the critical value of the fluid under consideration marks the onset of convection.

2. **Prandtl Number:**
   $$Pr = \frac{\mu}{\rho_0 \kappa}.$$
   Prandtl number is the ratio of kinematic viscosity of a fluid to thermal diffusivity of the fluid. Fluids with high values of Prandtl number are highly viscous and those with lower values of Prandtl number have higher thermal diffusivity. In case of non Non-Newtonian fluids, Prandtl number is very high.

3. **Cattaneo Number:**
   $$C_1 = \frac{\tau \kappa}{2d^2}.$$
   It is the ratio of relaxation time to characteristic time.

4. **Electric Rayleigh Number:**
   $$L = \frac{\epsilon_0 \left(e E_0 \Delta T d\right)^2}{\left(1 + \chi_e\right) \mu \kappa}.$$
   It is the ratio of electric force to viscous force.

5. **Couple Stress Parameter:**
   $$C = \frac{\acute{\mu}}{d^2 \mu} \left(0 \leq C \leq m\right).$$
   where, m is a finite positive real number.

Table 3.1: **Nomenclature**

d	distance between the plates
T	temperature
$\vec{q}$	velocity
p	pressure
μ	co-efficient of viscosity
$\acute{\mu}$	couple stress viscosity
ρ	density of the fluid
$\vec{g}$	acceleration due to gravity
$\vec{P}$	dielectric polarisation field
$\vec{E}$	electric field
$\left(\vec{P}.\bigtriangledown\right)\vec{E}$	polarisation force
$\vec{Q}$	heat flux
$\omega_1 = \frac{1}{2}\bigtriangledown \times\vec{q}$	spin
κ	thermal conductivity
α	co-efficient of thermal expansion
ϵ	amplitude
ϵ_0	electric permittivity of free space
ϵ_r	dielectric constant
χ_e	electric susceptibility
ΔT	Temperature difference between both the plates
ω	frequency
$\acute{\phi}$	perturbed electric scalar potential
l, m	wave numbers in two dimensional plane where $a^2 = l^2 + m^2$
φ	phase angle
$e = -\frac{\partial\epsilon_r}{\partial T}_{T=T_0}$	$\bigtriangledown_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$
$\bigtriangledown^2 = \bigtriangledown_1^2 + D^2$	$D = \frac{\partial}{\partial z}$, $K_1^2 = \pi^2 + a^2$

Chapter 4

The Effect of Temperature Modulation on the Onset of Rayleigh-B$\acute{e}$nard Convection in a Dielectric Couple Stress Fluid with Maxwell-Cattaneo Law

We study the effect of temperature modulation on the onset of Rayleigh-B$\acute{e}$nard Convection in a couple stress fluid in the presence of electric field with non-classical Maxwell-Cattaneo law.

A layer of couple stress fluid is considered between two infinite parallel surfaces and the surfaces are seperated by a distance d. Along z-axis, a uniform electric field is applied. A cartesian system is considered with origin at the lower limit and z-axis vertically upwards. Let the temperature difference between the two boundaries be denoted by ΔT.

The governing equations are:

Continuity Equation:

$$\bigtriangledown . \vec{q} = 0, \tag{4.1}$$

Conservation of Linear Momentum:

$$\rho_0 \left(\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\bigtriangledown)\vec{q} \right) = -\bigtriangledown p + \mu \bigtriangledown^2 \vec{q} - \acute{\mu} \bigtriangledown^4 \vec{q} + \rho \vec{g} + (\vec{P}.\bigtriangledown)\vec{E}, \tag{4.2}$$

Conservation of Energy:

$$\frac{\partial T}{\partial t} + (\vec{q}.\bigtriangledown)T = -\bigtriangledown . \vec{Q}, \tag{4.3}$$

Maxwell Cattaneo Heat Flux Equation:

$$\tau \left[\vec{Q} + \vec{\omega}_1 \times \vec{Q} \right] = -\vec{Q} - \kappa \bigtriangledown T, \tag{4.4}$$

Equation of state:

$$\rho = \rho_0 \left(1 - \alpha \left(T - T_0\right)\right), \tag{4.5}$$

Electrical Equation:

$$\bigtriangledown \times \vec{E} = 0, \tag{4.6}$$

$$\bigtriangledown \cdot \left(\epsilon_0 \vec{E} + \vec{P}\right) = 0, \vec{P} = \epsilon_0 \left(\epsilon_r - 1\right) \vec{E}, \tag{4.7}$$

Equation of state for dielectric constant:

$$\epsilon_r = \left(1 + \chi_e\right) - e \left(T - T_0\right), \tag{4.8}$$

The temperatures at the walls are dependent on time and are applied externally

$$T\left(0, t\right) = T_0 + \frac{1}{2}\Delta T \left[1 + \epsilon \cos \omega t\right], \tag{4.9}$$

and

$$T\left(d, t\right) = T_0 - \frac{1}{2}\Delta T \left[1 - \epsilon \cos\left(\omega t + \varphi\right)\right]. \tag{4.10}$$

We consider three cases for temperature modulation:
case 1:Symmetric($\varphi = 0$),
case 2:Asymmetric($\varphi = \pi$),
case 3:Only lower boundary is modulated($\varphi = -i\infty$).

4.1 Basic State

The fluid considered is assumed to be at rest and is characterized by

$$\vec{q_b} = 0, \rho = \rho_b\left(z, t\right), T = T_b\left(z, t\right), p = p_b\left(z, t\right), \vec{E} = \vec{E_b}\left(z\right), \vec{P} = \vec{P_b}\left(z\right) \tag{4.11}$$

On substituting equation (4.11) in the basic equations (4.1)-(4.8) we obtain,

$$\frac{\partial q_b}{\partial z} = -\rho_b g + P_b \frac{\partial P_b}{\partial z}, \tag{4.12}$$

$$\frac{\partial T_b}{\partial t} = -\frac{\partial Q_b}{\partial z}, \tag{4.13}$$

$$\frac{\partial Q_b}{\partial z} = -\kappa \frac{\partial^2 T_b}{\partial z^2}, \tag{4.14}$$

$$\left.\begin{aligned}\rho_b = \rho_0\left[1-\alpha\left(T_b-T_0\right)\right],\\ \epsilon_0\bigtriangledown.\vec{E}_b+\bigtriangledown.\vec{P}_b=0,\\ \epsilon_r=\left(1+\chi_e\right)-e\left(T_b-T_0\right),\\ \vec{E}_b=\left[\frac{\left(1+\chi_e\right)E_0}{\left(1+\chi_e\right)+\dfrac{e\Delta T}{d}z}\right]\acute{k},\\ \vec{P}_b=\epsilon_0E_0\left(1+\chi_e\right)\left[1-\frac{1}{\left(1+\chi_e\right)+\dfrac{e\Delta T}{d}z}\right]\acute{k}.\end{aligned}\right\} \tag{4.15}$$

Using equation (4.14) in equation (4.13), we get

$$\frac{\partial T_b}{\partial t}=\kappa\frac{\partial^2 T_b}{\partial z^2}, \tag{4.16}$$

The solution of equation (4.16) that satisfies the boundary conditions (4.9) and (4.10) is

$$T_b=T_0+\frac{\Delta T}{2}\left(1-\frac{2z}{d}\right)+\epsilon \boldsymbol{Re}\left\{\left[A\left(\lambda\right)e^{\dfrac{\lambda z}{d}}+A\left(-\lambda\right)e^{\dfrac{-\lambda z}{d}}\right]\right\}e^{-i\omega t}, \tag{4.17}$$

where,

$$\lambda=\left(1-i\right)\left(\frac{\omega d^2}{2\kappa}\right)^{\dfrac{1}{2}}, \tag{4.18}$$

$$A\left(\lambda\right)=\frac{\Delta T}{2}\left[\frac{e^{-i\varphi}-e^{-\lambda}}{e^{\lambda}-e^{-\lambda}}\right]$$

and *Re* represents the real part of the above equation.

4.2 Linear Stability Analysis

Let the fluid be disturbed by a perturbation of negligible magnitude.We have,

$$\vec{q}=\vec{q_b}+\vec{q}',\rho=\rho_b+\rho',p=p_b+p',T=T_b+T',\vec{P}=\vec{P_b}+\vec{P}',\vec{E}=\vec{E_b}+\vec{E}', \tag{4.19}$$

where, the quantities with infinitesimal perturbations are indicated by the prime and the suffix b represents the basic state.

The components of polarisation field and electric field after perturbation are considered to be $\left(P_1', \acute{P_2'}, \acute{P_3'}\right)$ and $(E_1', E_2', E_b\left(z\right)+E_3')$.

On linearisation, the second part of the equation (4.7) gives

$$P_i'=\epsilon_0\chi_e E_i'\; for\; i=1,2. \tag{4.20}$$

$$P'_3 = \epsilon_0 \chi_e E'_3 - e\epsilon_0 E_0 T'. \tag{4.21}$$

Equation (4.6) implies that $\vec{E'} = \bigtriangledown \phi'$.

Using equation (4.19) in the governing equations (4.1)-(4.8), we get

$$\bigtriangledown . \vec{q'} = 0, \tag{4.22}$$

$$\rho_0 \frac{\partial \vec{q'}}{\partial t} = - \bigtriangledown p' + \mu \bigtriangledown^2 \vec{q'} - \mu' \bigtriangledown^4 \vec{q'} - \rho' g \hat{k} + P_b . \bigtriangledown \vec{E'} + \vec{P'} . \bigtriangledown E_b, \tag{4.23}$$

$$\frac{\partial T'}{\partial t} + w' \frac{\partial T_b}{\partial z} = - \bigtriangledown . \vec{Q'}, \tag{4.24}$$

$$\left(1 + \tau \frac{\partial}{\partial t}\right) \vec{Q'} = \frac{\tau\kappa}{2} \frac{\partial T_b}{\partial z} \left(\frac{\partial \vec{q'}}{\partial z} - \bigtriangledown w'\right) - \kappa \bigtriangledown T', \tag{4.25}$$

Operating divergence on the equation (4.25) and using equation (4.24), we get

$$\left(1 + \tau \frac{\partial}{\partial t}\right) \frac{\partial T'}{\partial t} + \left(1 + \tau \frac{\partial}{\partial t}\right) w' \frac{\partial T_b}{\partial z} = \frac{\tau\kappa}{2} \frac{\partial T_b}{\partial z} \bigtriangledown^2 w' + \kappa \bigtriangledown^2 T',$$

$$\rho' = -\rho_0 \alpha T', \tag{4.26}$$

$$\epsilon' = -\epsilon_0 e T',$$

$$\bigtriangledown . \left(\epsilon_0 \vec{E'} + \vec{P'}\right) = 0, \tag{4.27}$$

$$\epsilon_0 \frac{\partial E_b}{\partial z} + \epsilon_0 \bigtriangledown . \vec{E'} + \frac{\partial P_b}{\partial z} + \bigtriangledown . \vec{P'} = 0.$$

Substituting equation (4.26) in equation (4.23),

$$\rho_0 \frac{\partial \vec{q'}}{\partial t} = - \bigtriangledown p' + \mu \bigtriangledown^2 \vec{q'} - \mu' \bigtriangledown^4 \vec{q'} + \rho_0 \alpha T' g \hat{k} + P_b . \bigtriangledown \vec{E'} + \vec{P'} . \bigtriangledown E_b, \tag{4.28}$$

Operating curl twice on equation (4.28) to eliminate pressure and introducing the electric potential $\acute{\phi}$ in the resulting equation, we get

$$\rho_0\frac{\partial}{\partial t}\left(\bigtriangledown^2 w'\right)=\mu\bigtriangledown^4 w'-\mu'\bigtriangledown^6 w'+\rho_0\alpha g\bigtriangledown_1^2\vec{T}'+\frac{\epsilon_0 e^2E_0^2\Delta T\bigtriangledown_1^2 T}{1+\chi_e}-\frac{\Delta T}{d}\epsilon_0 eE_0\bigtriangledown_1^2 D\phi', \quad (4.29)$$

Also,

$$\left(1+\chi_e\right)\bigtriangledown^2\phi'-eE_0DT'=0. \quad (4.30)$$

The resulting equations after perturbation (4.2),(4.29) and (4.30) are non dimensionalised employing the following scaling parameters

$$\left(x^*,y^*,z^*\right)=\left(\frac{x'}{d},\frac{y'}{d},\frac{z'}{d}\right),t^*=\frac{t'}{d^2/\kappa},w^*=\frac{w'}{\kappa/d},T^*=\frac{T'}{\Delta T},$$
$$\phi^*=\frac{\phi'}{eE_0d\Delta T/\left(1+\chi_e\right)}, \quad (4.31)$$

After dropping the asterisk, we obtain

$$\frac{1}{Pr}\frac{\partial}{\partial t}\left(\bigtriangledown^2 w\right)-\boldsymbol{L}\bigtriangledown_1^2 T+\boldsymbol{L}\frac{\partial}{\partial z}\left(\bigtriangledown_1^2\phi\right)=\boldsymbol{R}\bigtriangledown_1^2 T+\bigtriangledown^4 w-\boldsymbol{C}\bigtriangledown^6 w, \quad (4.32)$$

$$\left(1+2\boldsymbol{C}_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t}+\left(1+2\boldsymbol{C}_1\frac{\partial}{\partial t}\right)w\frac{\partial T_0}{\partial z}=\boldsymbol{C}_1\bigtriangledown^2 w\frac{\partial T_0}{\partial z}+\bigtriangledown^2 T, \quad (4.33)$$

$$\bigtriangledown^2\phi-\frac{\partial T}{\partial z}=0. \quad (4.34)$$

The non-dimensional parameters are given as:

$Pr=\dfrac{\mu}{\rho_0\kappa}$ (Prandtl Number)

$\boldsymbol{L}=\dfrac{\epsilon_0\left(eE_0\Delta Td\right)^2}{\left(1+\chi_e\right)\mu\kappa}$ (Electric Rayleigh Number)

$\boldsymbol{R}=\dfrac{\rho_0\alpha g d^3\Delta T}{\mu\kappa}$ (Rayleigh Number)

$\boldsymbol{C}=\dfrac{\acute{\mu}}{d^2\mu}$ (Couple Stress Parameter)

$\boldsymbol{C}_1=\dfrac{\tau\kappa}{2d^2}$ (Cattaneo Number)

After non dimensionalisation, $\dfrac{\partial T_b}{\partial z}$ becomes $\dfrac{\partial T_0}{\partial z}$, where

$$\frac{\partial T_0}{\partial z}=-1+\epsilon f\left(z\right), \quad (4.35)$$

$$f(z) = \boldsymbol{Re}\left\{\left[A(\lambda)\, e^{\lambda z} + A(-\lambda)\, e^{-\lambda z}\right] e^{-i\omega t}\right\},$$

$$and\ A(\lambda) = \frac{\lambda}{2}\left[\frac{e^{-i\varphi} - e^{-\lambda}}{e^{\lambda} - e^{-\lambda}}\right].$$

Equations (4.32) to (4.34) are solved with respect to the conditions:

$$w = \frac{\partial^2 w}{\partial z^2} = T = \frac{\partial \phi}{\partial z} = 0\ at\ z = 0, 1 \tag{4.36}$$

T and ϕ are eliminated from equations (4.32) to (4.34), we get a differential equation of the form:

$$\left\{\left[\frac{1}{Pr}\frac{\partial}{\partial t} - \nabla^2 + \boldsymbol{C}\nabla^4\right]\left[\left(1 + 2C_1\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t} - \nabla^2\right]\nabla^4\right\} w$$
$$= \left\{\left[\boldsymbol{L}\,\nabla_1^2\,\nabla^2 - \boldsymbol{L}\,\nabla_1^2\,\frac{\partial^2}{\partial z^2} + \boldsymbol{R}\,\nabla_1^2\,\nabla^2\right]\left[C_1\,\nabla^2\,\frac{\partial T_0}{\partial z} - \left(1 + 2C_1\frac{\partial}{\partial t}\right)\frac{\partial T_0}{\partial z}\right]\right\} w. \tag{4.37}$$

The boundary conditions on w for solving equation (4.37) in the absence of dimension is:

$$w = \frac{\partial^2 w}{\partial z^2} = \frac{\partial^4 w}{\partial z^4} = \frac{\partial^6 w}{\partial z^6} = \frac{\partial^8 w}{\partial z^8} = 0\ at\ z = 0, 1.$$

4.3 Method of Solution

For the basic temperature distribution (4.35), the eigenvalue *R* and eigenfunction w in (4.37) departs from the temperature profile $\frac{\partial T_0}{\partial z} = -1$ by values of order ϵ. The eigenvalues and eigenfunction in equation (4.37) is expanded in the form:

$$(\boldsymbol{R}, w) = (\boldsymbol{R}_0, w_0) + \epsilon\,(\boldsymbol{R}_1, w_1) + \epsilon^2\,(\boldsymbol{R}_2, w_2) + ...$$

The above expansion is substituted in equation (4.37). A following equations are obtained by equating the coefficients of like powers of ϵ

$$L_1 w_0 = 0. \tag{4.38}$$

$$L_1 w_1 = \left\{ \left[C_1 \bigtriangledown^2 - \left(1 + 2C_1 \frac{\partial}{\partial t} \right) \right] \left[L \bigtriangledown_1^2 \bigtriangledown^2 - L \bigtriangledown_1^2 \frac{\partial^2}{\partial z^2} + R_0 \bigtriangledown_1^2 \bigtriangledown^2 \right] \right\} f w_0 + \left[-C_1 \bigtriangledown^2 + \left(1 + 2C_1 \frac{\partial}{\partial t} \right) \right] \bigtriangledown^2 \bigtriangledown_1^2 R_1 w_0. \quad (4.39)$$

$$L_1 w_2 = \left[C_1 \bigtriangledown^2 f - \left(1 + 2C_1 \frac{\partial}{\partial t} \right) f \right] \left\{ \left[L \bigtriangledown_1^2 \bigtriangledown^2 - L \bigtriangledown_1^2 \frac{\partial^2}{\partial z^2} \right] w_1 + [R_0 w_1 + R_1 w_0] \bigtriangledown_1^2 \bigtriangledown^2 + \left[C_1 \bigtriangledown^2 - \left(1 + 2C_1 \frac{\partial}{\partial t} \right) \right] \bigtriangledown_1^2 \bigtriangledown^2 [R_1 w_1 + R_2 w_0] \right. . \quad (4.40)$$

where

$$L_1 = \left[\frac{1}{Pr} \frac{\partial}{\partial t} - \bigtriangledown^2 + C \bigtriangledown^4 \right] \left[\left(1 + 2C_1 \frac{\partial}{\partial t} \right) \frac{\partial}{\partial t} - \bigtriangledown^2 \right] \bigtriangledown^4 - \left[1 + 2C_1 \frac{\partial}{\partial t} - C_1 \bigtriangledown^2 \right] \bigtriangledown_1^2 \left[L \left(\bigtriangledown^2 - \frac{\partial^2}{\partial z^2} \right) + R_0 \bigtriangledown^2 \right] . \quad (4.41)$$

4.3.1 Solution to Zeroth Order Problem

The zeroth order problem is the Rayleigh B$\acute{e}$nard problem in the absence of thermal modulation. The velocity perturbation in the vertical direction w_0 which represents the minimal mode of convection is given by:

$$w_0 = \sin(\pi z) \exp[i(lx + my)] . \quad (4.42)$$

Substituting equation (4.42) into equation (4.38) we get,

$$R_0 = \frac{K_1^6 K_1^2 (1 + C K_1^2) + L (\pi^2 - K_1^2) a^2 (1 + C_1 K_1^2)}{K_1^2 a^2 (1 + C_1 K_1^2)}, \quad (4.43)$$

where $K_1^2 = \pi^2 + a^2$.

In equation (4.43), if $L = 0,\ C = 0\ and\ C_1 = 0$, we get the expression for classical Rayleigh problem.

If $L = 0\ and\ C_1 = 0$, the expression (4.43) reduces to the expression obtained by Siddheshwar and Pranesh[54].

If $C = 0$, the expression (4.43) reduces to the expression obtained by Pranesh and Kiran[38].

4.3.2 Solution to First Order Problem

We have

$$L_1 w_1 = \left\{ \left[\left[-1 - C_1 K_1^2 \right] \left[La^2 K_1^2 - La^2 \pi^2 + a^2 K_1^2 R_0 \right] \right] f + \left[1 + C_1 K_1^2 \right] a^2 K_1^2 R_1 \right\} w_0. \tag{4.44}$$

For the existence of a solution for the above equation, the orthogonality condition must hold for the null space of the operator L_1 and the right hand part of the equation (4.44) must be independent of time. The only consistent term that appears on the right hand part of equation (4.44) is $R_1 \bigtriangledown^2 \bigtriangledown_1^2 \sin(\pi z)$ due to the sinusoidal variation of f and hence $R_1 = 0$. Thus it is implied that all the odd coefficients $R_1 = R_3 = ...$ are zero in equation (4.39).

Fourier series expansion is applied and the right hand side of the equation is expanded to solve equation(4.44). w_1 is obtained by inverting the operator L_1 term wise and we get:

$$w_1 = -A_1 \left(La^4 + R_0 a^2 K_1^2 \right) Re \left\{ \sum \frac{Bn(\lambda)}{L_1(\omega)} e^{-i\omega t} \sin(\pi z) \right\}, \tag{4.45}$$

where

$$A_1 = \left(1 + C_1 K_1^2 \right),$$

$$B_n(\lambda) = A(\lambda) g_{n1}(\lambda) A(-\lambda) g_{n1}(-\lambda) \tag{4.46}$$

$$= \frac{(2n\pi^2\lambda^2) \left[\left(e^{-\lambda} - e^{\lambda} \right) + (-1)^n \left(e^{-\lambda - i\phi} - e^{\lambda - i\phi} \right) \right]}{\left(e^{\lambda} - e^{-\lambda} \right) \left[\lambda^2 + (n+1)^2 \pi^2 \right] \left[\lambda^2 + (n-1)^2 \pi^2 \right]}, \tag{4.47}$$

$$L_1(\omega) = \left[\left(K_1^2 + CK_1^4 \right) \left(K_1^2 - 2C_1\omega^2 \right) - \frac{\omega^2}{Pr} \right] K_1^4 + \left(-La^2 - R_0 K_1^2 \right) a^2 \left(1 + C_1 K_1^2 \right) - i\omega \left\{ \left[\left(K_1^2 + CK_1^4 \right) + \frac{1}{Pr} \left(K_1^2 - 2C_1\omega^2 \right) \right] K_1^4 + 2C_1 a^2 \left(-La^2 - R_0 K_1^2 \right) \right\}. \tag{4.48}$$

The equation for w_2 becomes,

$$L_1 w_2 = A_1 R_2 a^2 K_1^2 w_0 - A_2 \left[La^2 + K_1^2 R_0\right] a^2 f w_1 \tag{4.49}$$

where

$$A_2 = C_1 K_1^2 + 1 - i\omega \left(2C_1\right) \tag{4.50}$$

Equation (4.49) shall not be solved, but is used in determining R_2. Thus by using Venezian[64] method, the expression for correction Rayleigh number is obtained as:

$$R_2 = -\frac{\left(La^2 + K_1^2 R_0\right)^2 a^2}{2K_1^2} Re \sum \frac{\left|B_n\left(\lambda\right)\right|^2 \left|A_2\right|^2}{\left|L_2\left(\omega\right)\right|^2} \left[\frac{L_2\left(\omega\right) + L_2^*\left(\omega\right)}{2}\right], \tag{4.51}$$

where,

$$L_2\left(\omega\right) = L_1\left(\omega\right) A_2^*.$$

A_2^* and $L_2^*\left(\omega\right)$ are the conjugates of A_2 and $L_2\left(\omega\right)$.

4.3.3 Minimum Rayleigh Number for Convection

The Rayleigh number R, eigenvalue corresponding to the eigen function w is obtained will remain time bounded. R is expressed in terms of horizontal wave number a and amplitude ϵ and hence we write

$$R\left(a, \epsilon\right) = R_0\left(a\right) + \epsilon^2 R_2\left(a\right) + ... \tag{4.52}$$

The critical value of thermal Rayleigh number is calculated up to $O\left(\epsilon^2\right)$, by computing the values R_0 and R_2 at $a = a_0$. When R_4 is to be evaluated, a_2 is taken into account where R_2 is minimized by $a = a_2$. The critical value of R_2 expressed as R_{2c} is evaluated by replacing $a = a_0$ in the expression of R_2 where a_0 is the value at which the minimum value of R_0 given by (4.43) is obtained.

Evaluation of R_{2c} is done for three cases:

Case 1: The wall temperatures are modulates symmetrically with phase angle $\varphi = 0$. In this case $B_n\left(\lambda\right) = b_n \textit{ or } 0$ corresponding to even or odd values of n respectively.

Case 2: The wall temperatures are modulated asymmetrically with the phase angle $\varphi = \pi$. In this case $B_n\left(\lambda\right) = 0 \textit{ or } b_n$ corresponding to even or odd values of n respectively.

Case 3: Only the lower wall is modulated and the temperature of the upper plate remains constant with phase angle $\varphi = i\infty$. Here $B_n\left(\lambda\right) = \dfrac{b_n}{2}$ for all integer values of n.

where,

$$b_n = \frac{4n\pi^2\lambda^2}{\left[\lambda^2 + (n+1)^2\pi^2\right]\left[\lambda^2 + (n-1)^2\pi^2\right]}. \tag{4.53}$$

In dimensionless form equation (4.18) can be written as:

$$\lambda = (1-i)\left(\frac{\omega}{2}\right)^{\frac{1}{2}}, \tag{4.54}$$

and hence

$$|b_n|^2 = \frac{16n^2\pi^2\omega^2}{\left[\omega^2 + (n+1)^4\pi^4\right]\left[\omega^2 + (n-1)^4\pi^4\right]}. \tag{4.55}$$

Using equation (4.55) for $B_n(\lambda)$ in (4.51), the expression for R_{2c} is obtained as:

$$R_{2c} = -\frac{\left(La^2 + K_1^2 R_0\right)^2 a^2}{2K_1^2} Re \sum \frac{|B_n(\lambda)|^2 |A_2|^2}{|L_2(\omega)|^2}\left[\frac{L_2(\omega) + L_2^*(\omega)}{2}\right]. \tag{4.56}$$

In equation (4.56), the summation remains valid for even values of n in first case, odd values of n in second case and all values of n in the third case. In all three instances, the infinite series given in equation (4.56) converges quickly.

Chapter 5

Results, Discussions and Conclusions

The effect of thermal modulation on the onset of convection in a couple stress fluid in the presence of electric field is examined by making use of linear stability analysis. The expression that depicts the critical value of correction Rayleigh number is evaluated in terms of frequency of modulation. R_{2c} has been evaluated in three cases: Case 1: Symmetric temperature field with the phase angle $\varphi = 0$. Case 2: Asymmetric temperature field with phase angle $\varphi = \pi$.Case 3: When modulation is applied to the temperature of the lower wall only with $\varphi = i\infty$.

We assume that the amplitude of modulation is very small when compared to the steady difference in temperature that has been imposed on the system. The results obtained are validated depending on the value of frequency ω. If the value of frequency is small, the period of modulation becomes large and hence the disturbance grows to a greater extend. For large values of frequency, R_{2c} tends to zero that is the effect of modulation becomes negligible. As a result, only moderate values of frequency are taken into consideration in this study.

The results has been presented in the figures.The effect of R_{2c} on the stability of the system is given by the sign of R_{2c}. A positive R_{2c} implies that it stabilizes the system.

The effect of symmetric temperature field on the convection in a couple stress fluid for various parameters are depicted in figures 5.2-5.5. Figure 5.2 is the plot of correction Rayleigh number versus frequency for various values of couple stress parameter C. From the figure we observe that increase in C increases the value of R_{2c}. Since the fluid contains suspended particles, increase in couple stress parameter increases the viscosity of the fluid thereby stabilizing the system.

Figure 5.3 shows the plot of correction Rayleigh number versus frequency for various values of electric Rayleigh number L. We observe that as L increases, R_{2c} also increases. The Electric Rayleigh number L given by the ratio of polarisation electric force to gravitational force stabilizes the system.

The plot of correction Rayleigh number versus frequency for various values of Prandtl number

Pr is shown in figure 5.4. It is observed that R_{2c} decreases with increase in Pr. Therefore we can conclude that increase in the viscosity of the fluid destabilizes the system.

The plot of correction Rayleigh number versus frequency for various values of Cattaneo number C_1 is depicted in figure 5.5. It is observed that increase in Cattaneo number decreases R_{2c}. Thus C_1 has a destabilizing influence on the system.

From the above figures we also observed that for small values of ω, say up to ω_c, R_{2c} increases and there after it decreases that is when $\omega{<}\omega_c$, the system is stable and when $\omega{>}\omega_c$, the system is unstable. For large values of ω, R_{2c} becomes zero.

Figures 5.6-5.9 and 5.10-5.13 respectively are the plots for asymmetric modulation and lower wall modulation for various parameters.We observe that the results in these cases are quantitatively similar to that of symmetric modulation.

Following are the conclusions drawn from the study.

1. System is most stable when the temperature field is asymmetric.
2. Symmetric temperature field and lower wall modulation leads to sub-critical motion.
3. Modulation disappears in case of large frequency.
4. The results of the study shows that the convection in couple stress fluid with Maxwell-Cattaneo law can be controlled.
5. Maxwell-cattaneo law involve a wave type of heat propagation and does not suffer from the unphysical result of infinite transport of heat.The Fourier law over predicts the critical Rayleigh number when compared to that predicted by non-classical law.Over stability is the preferred mode of convection.

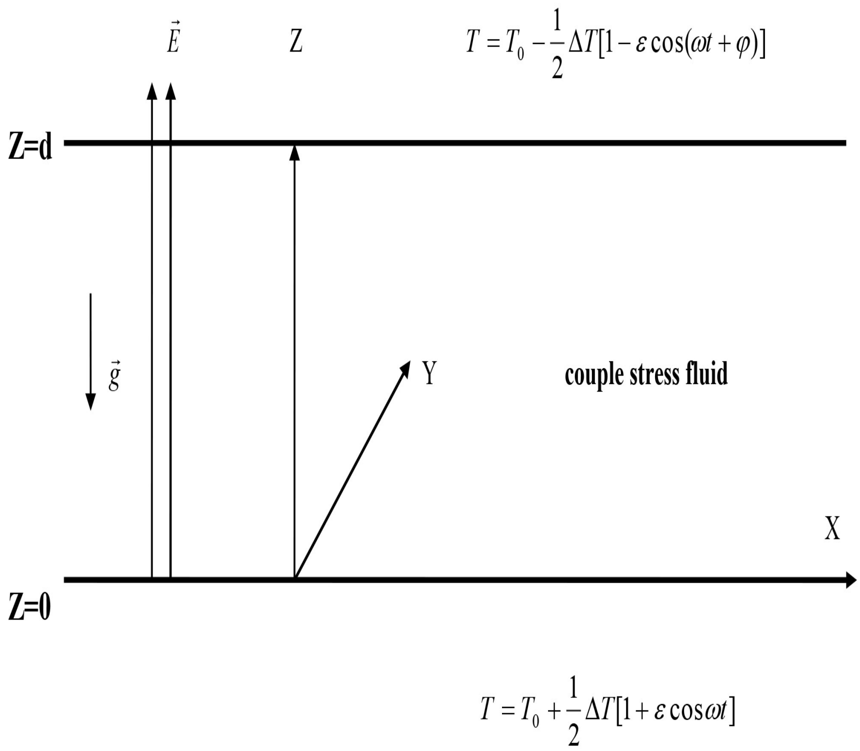

Figure 5.1: Physical Configuration

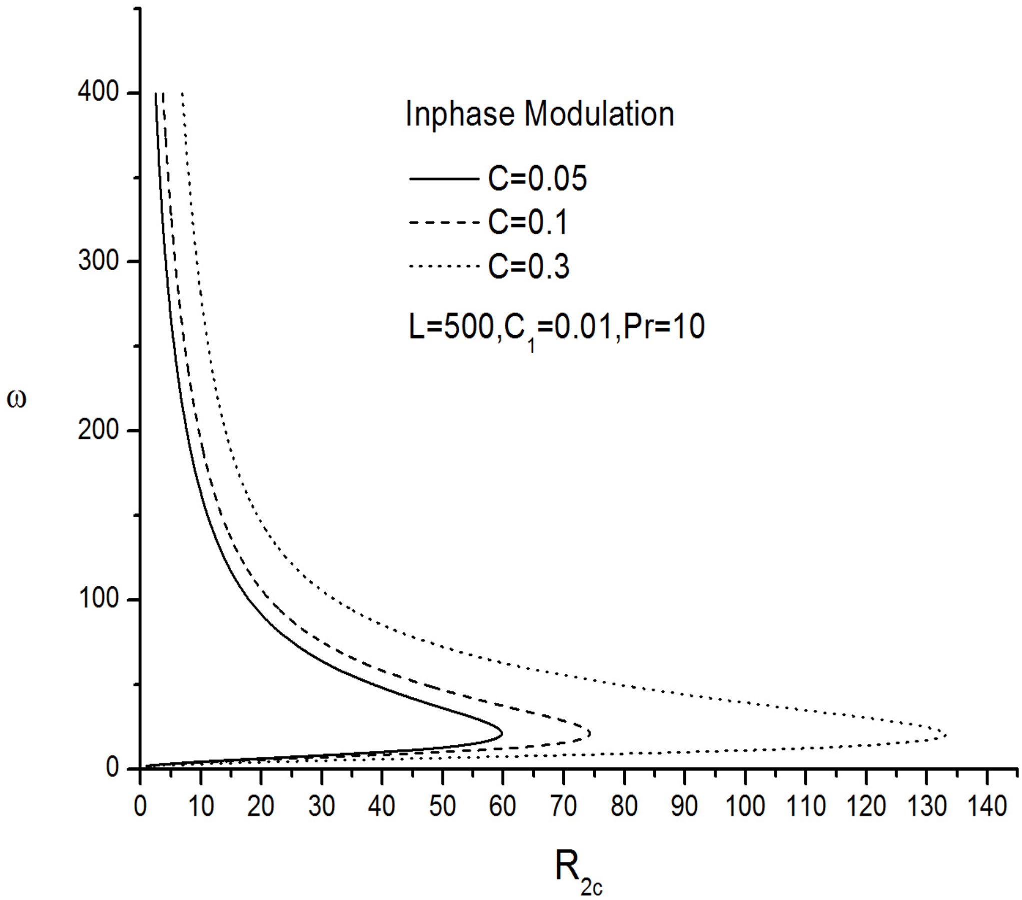

Figure 5.2: Plot of correction Rayleigh number R_{2c} versus frequency ω for various values of couple stress parameter C for in phase modulation.

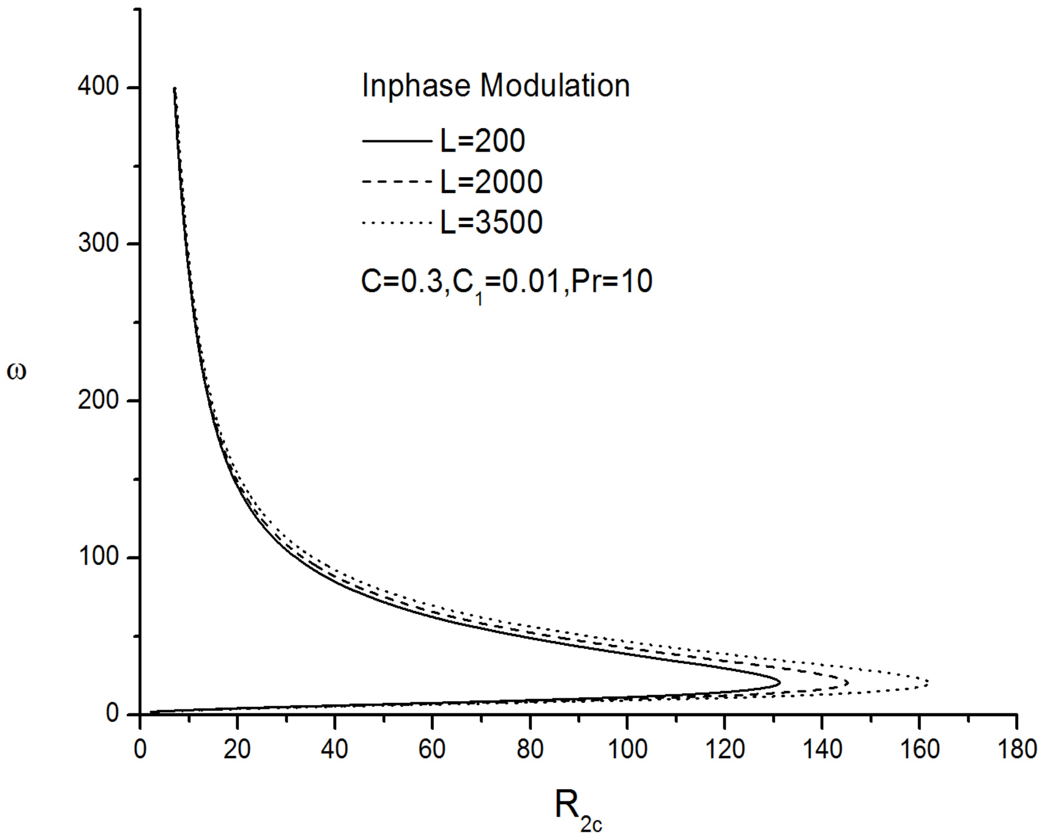

Figure 5.3: Plot of correction Rayleigh number R_{2c} versus frequency ω for various values of electric Rayleigh number L for in phase modulation.

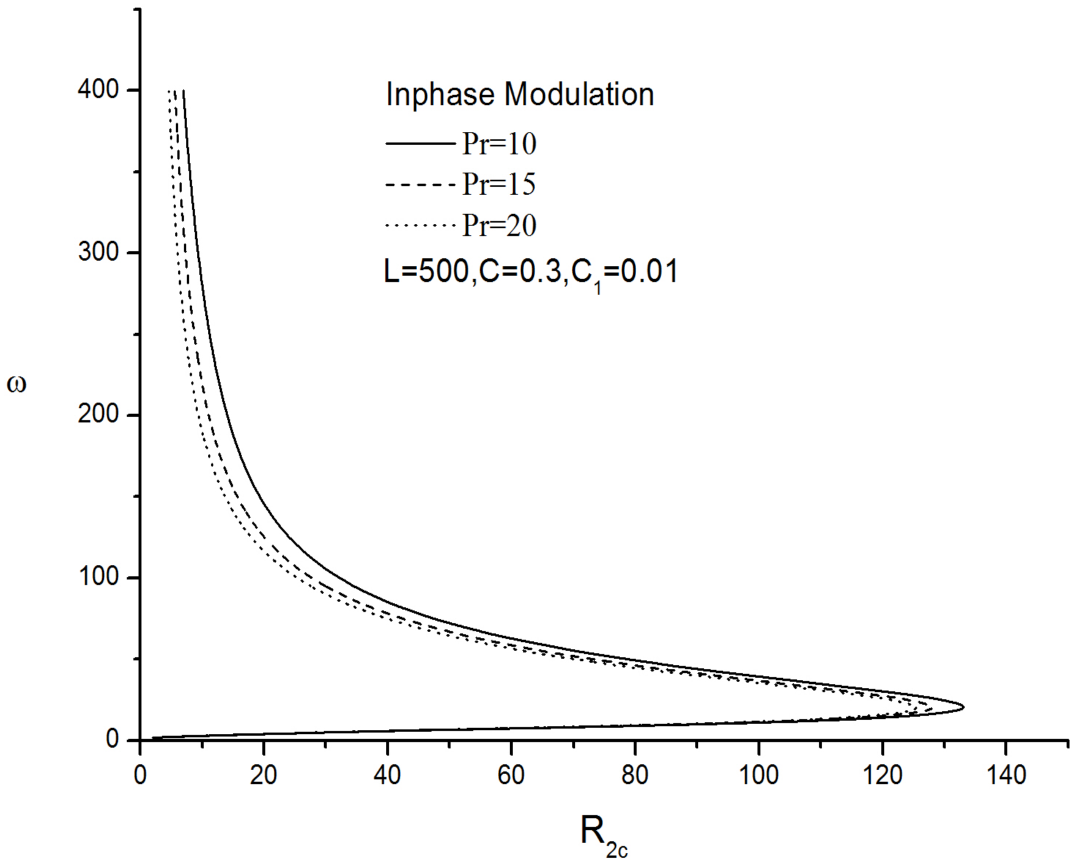

Figure 5.4: Plot of correction Rayleigh number R_{2c} versus frequency ω for various values of Prandtl number Pr for in phase modulation.

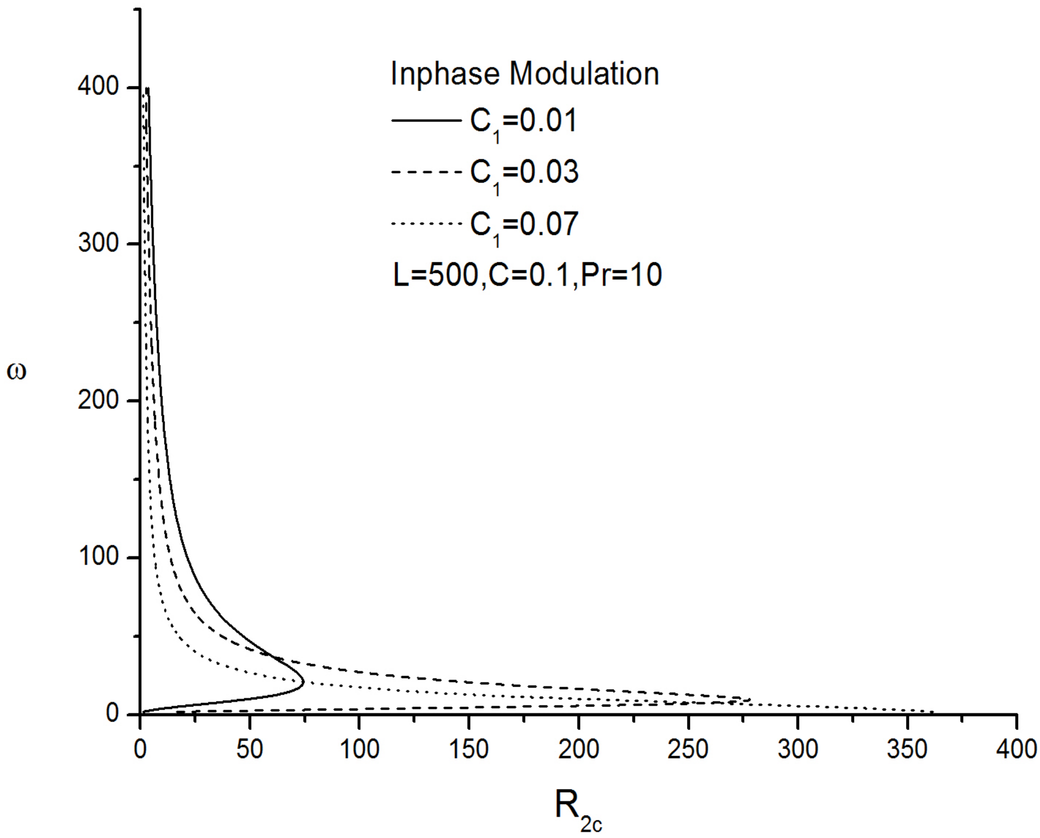

Figure 5.5: Plot of correction Rayleigh number R_{2c} versus frequency ω for various values of Cattaneo number C_1 for in phase modulation.

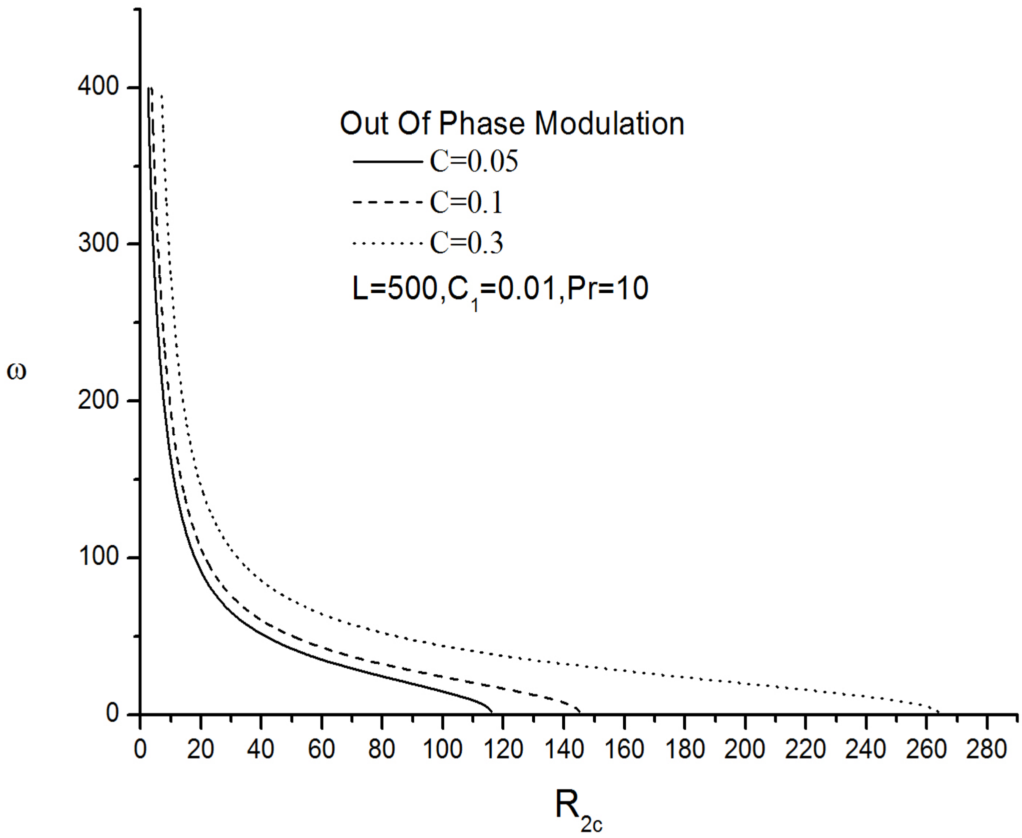

Figure 5.6: Plot of correction Rayleigh number R_{2c} versus frequency ω for various values of couple stress parameter C for out of phase modulation.

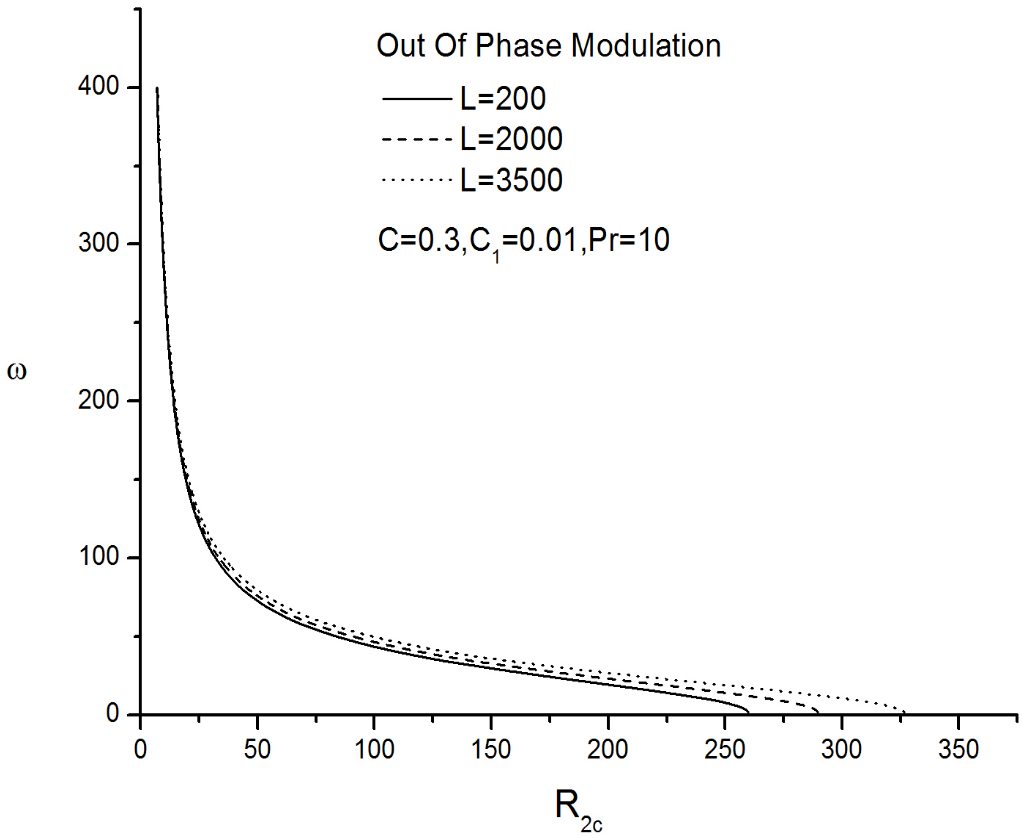

Figure 5.7: Plot of correction Rayleigh number R_{2c} versus frequency ω for various values of electric Rayleigh number L for out of phase modulation.

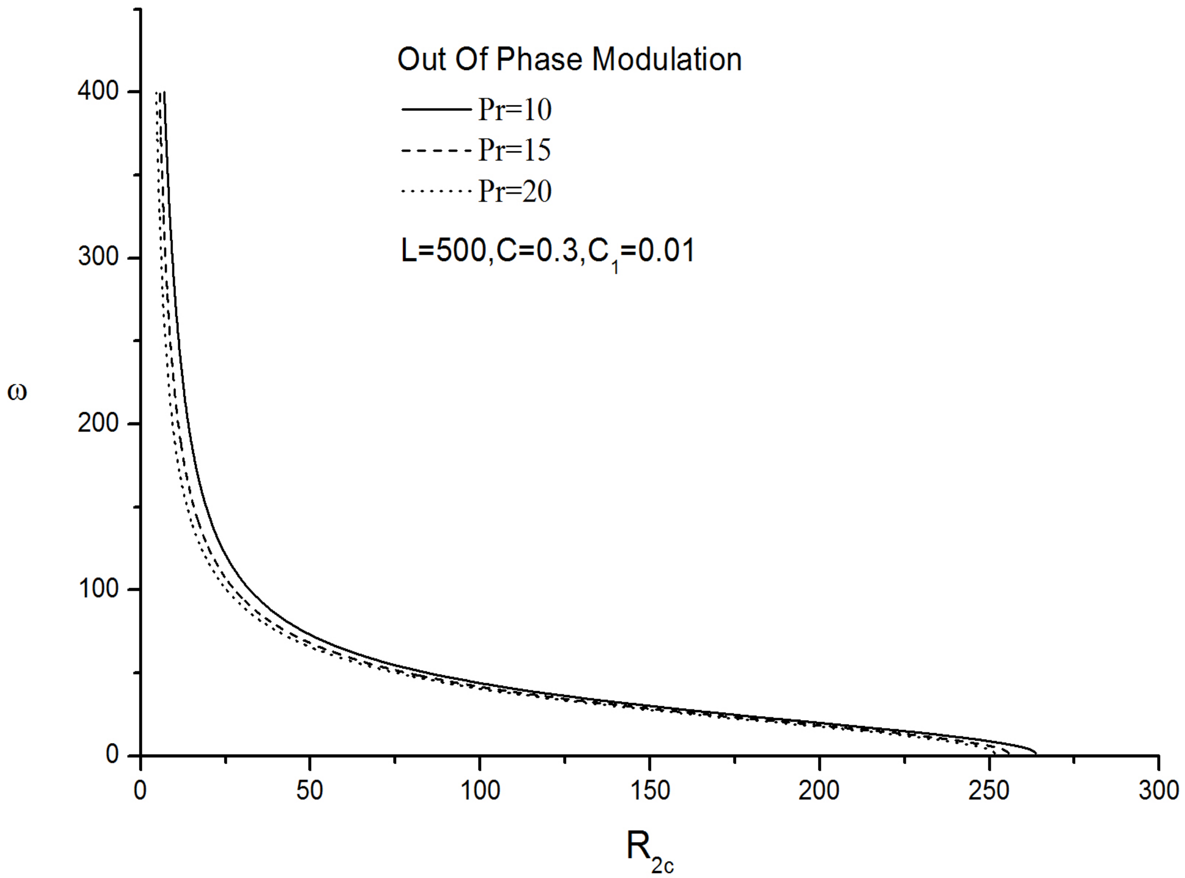

Figure 5.8: Plot of correction Rayleigh number R_{2c} versus frequency ω for various values of Prandtl number Pr for out of phase modulation.

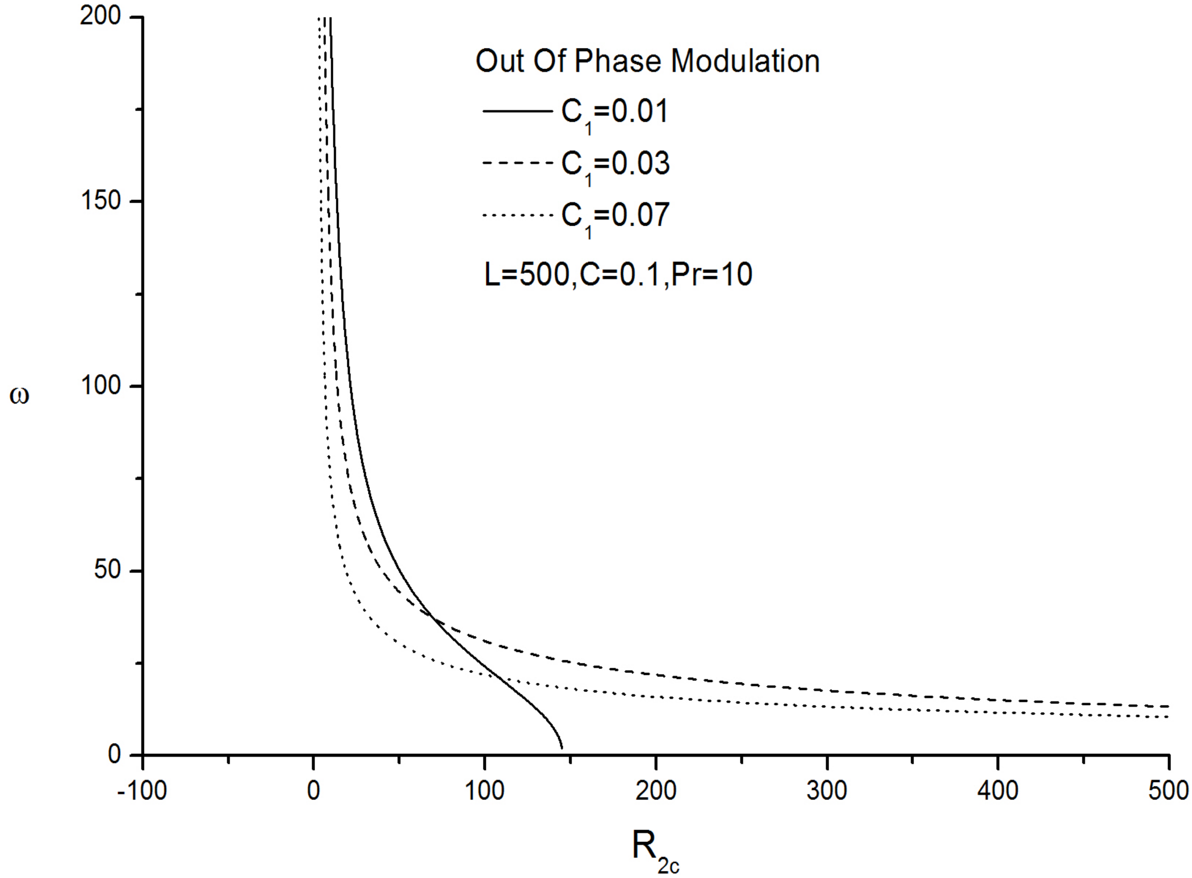

Figure 5.9: Plot of correction Rayleigh number R_{2c} versus frequency ω for various values of Cattaneo number C_1 for out of phase modulation.

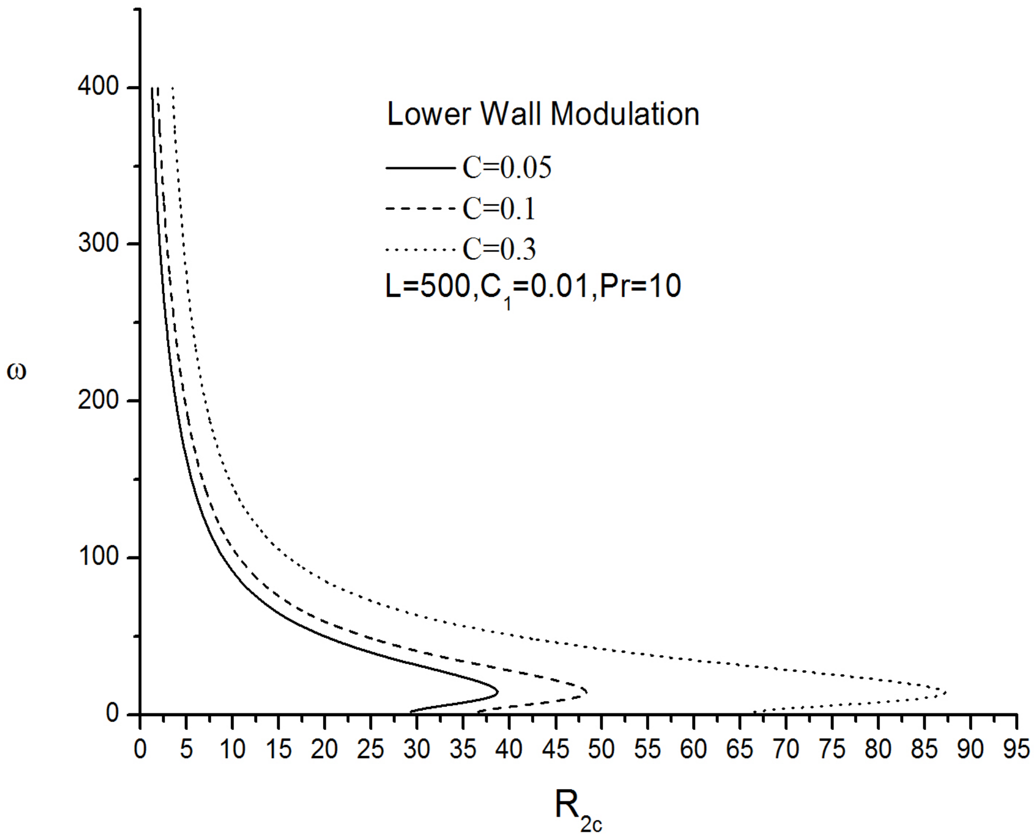

Figure 5.10: Plot of correction Rayleigh number R_{2c} versus frequency ω for various values of couple stress parameter C for lower wall modulation.

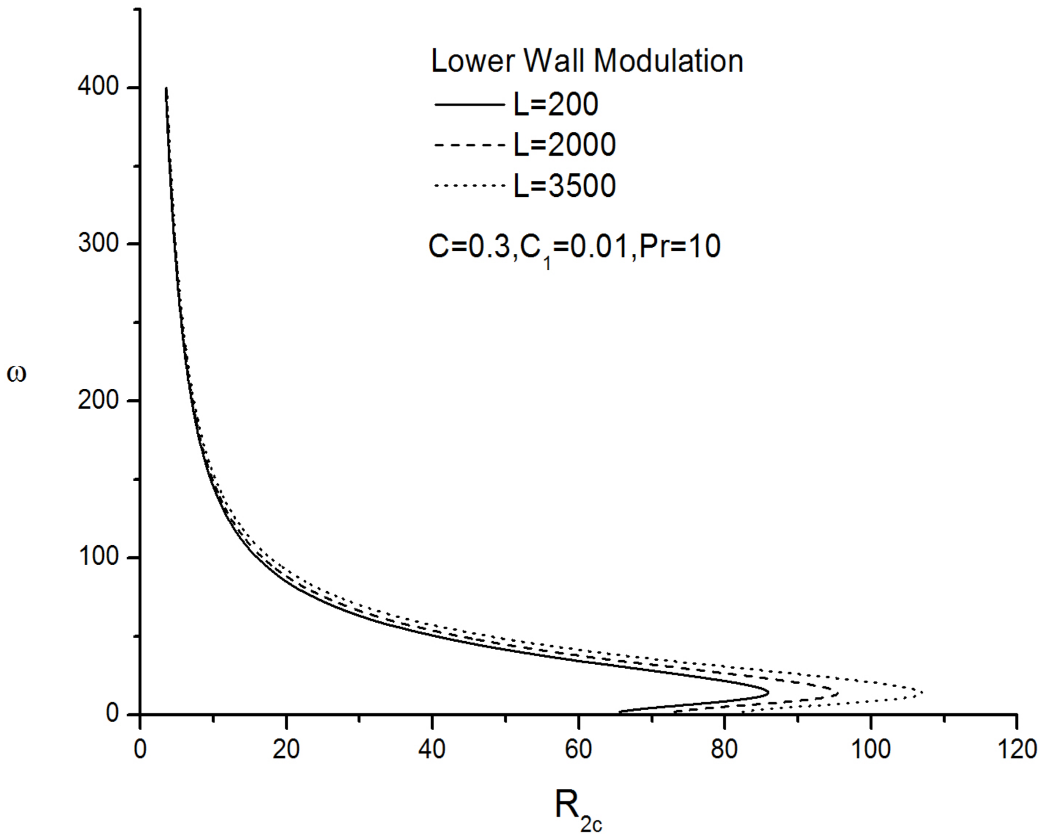

Figure 5.11: Plot of correction Rayleigh number R_{2c} versus frequency ω for various values of electric Rayleigh number L for lower wall modulation.

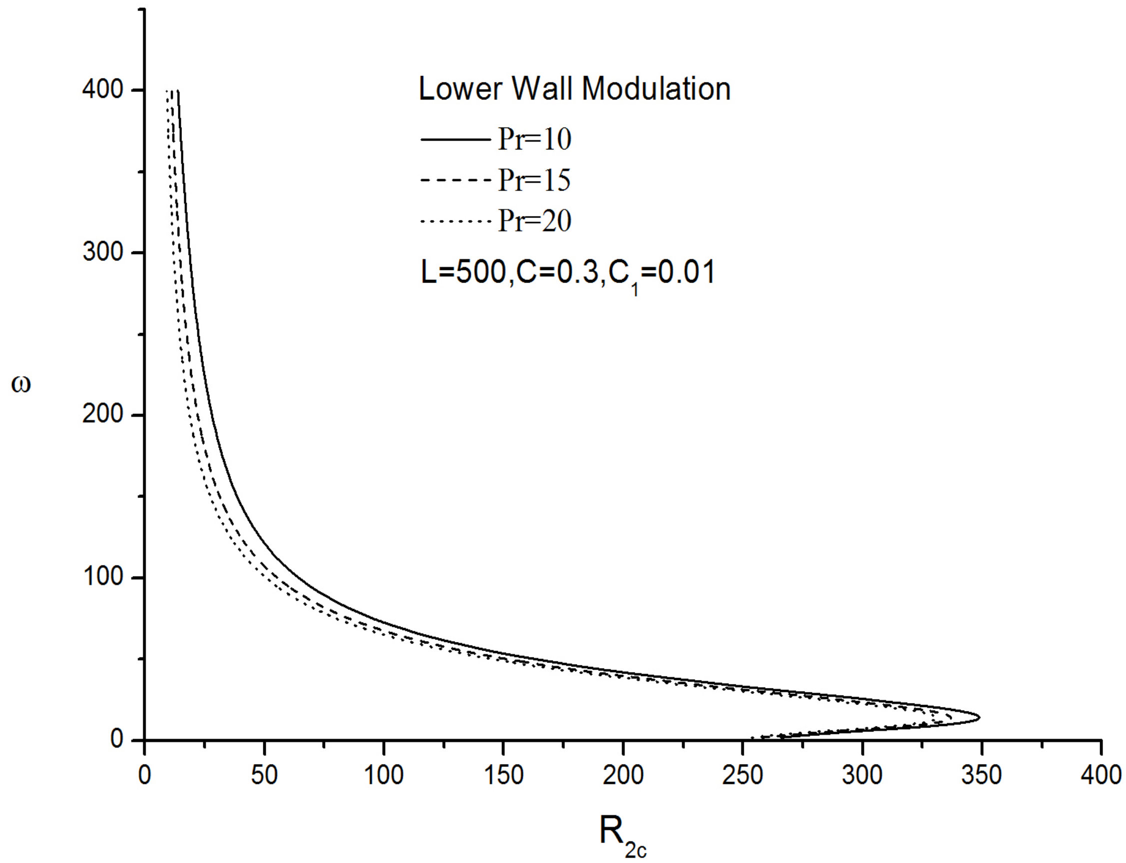

Figure 5.12: Plot of correction Rayleigh number R_{2c} versus frequency ω for various values of Prandtl number Pr for lower wall modulation.

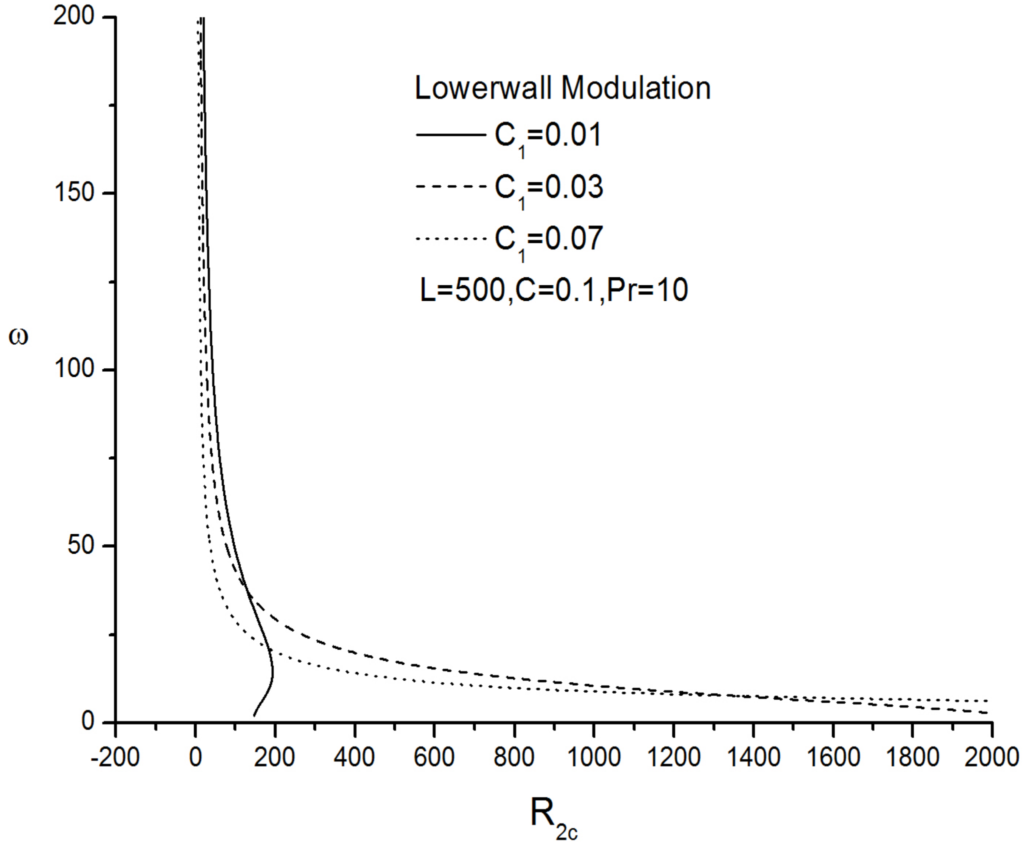

Figure 5.13: Plot of correction Rayleigh number R_{2c} versus frequency ω for various values of Cattaneo number C_1 for lower wall modulation.

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